Physics of ultracold Bose gases in one-dimension and solitons

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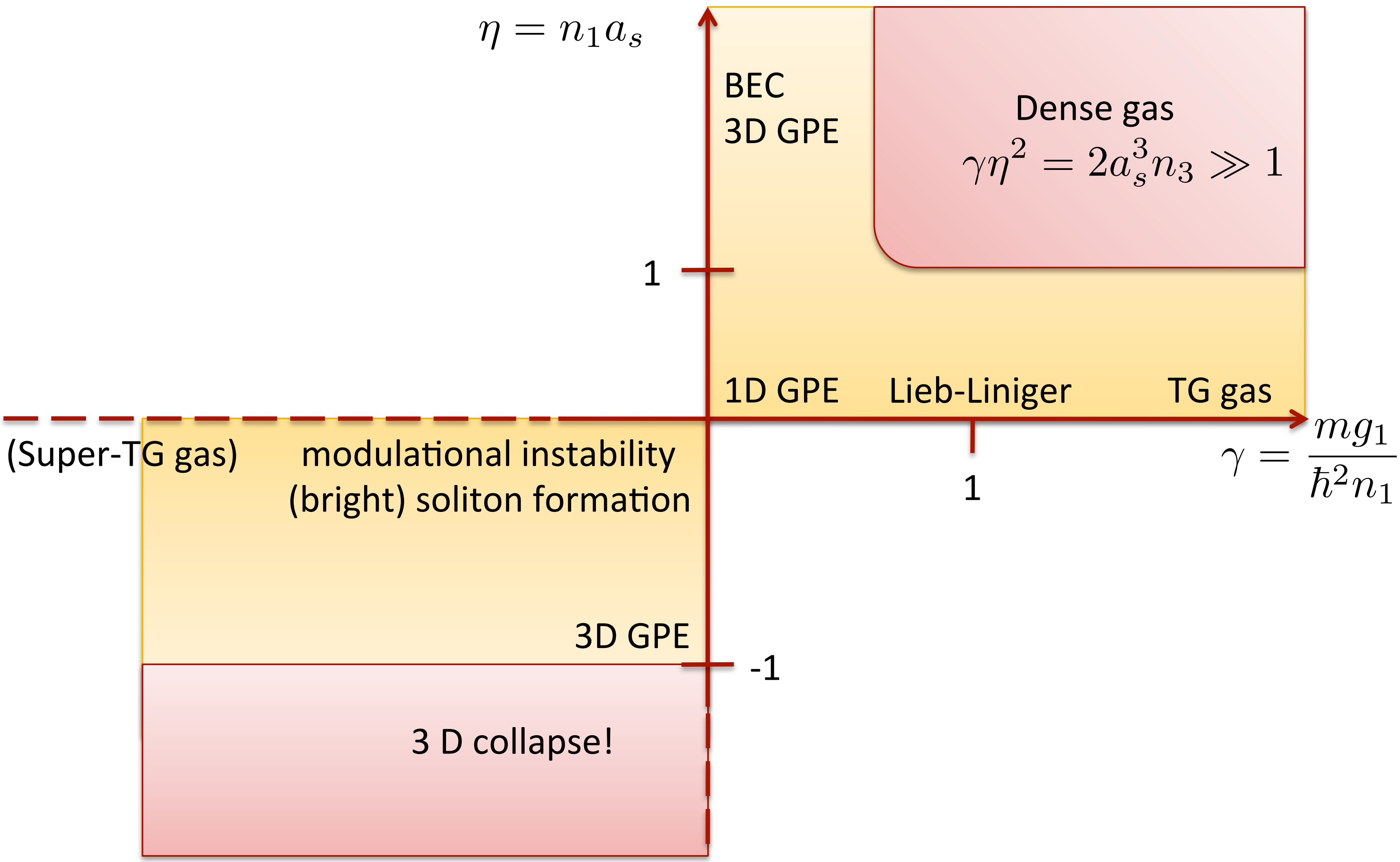
One dimension is different!

- To be covered in these lectures:
  - Introduction to solitons
  - Absence of true Bose-Einstein condensation
  - Strongly-correlated many-body physics with a dilute gas
  - Attractive bosons and quantum bright solitons
  - Bosons play fermions: Lieb-Liniger model
  - Superfluid or not superfluid (or maybe both?)
  - Where are solitons in the Lieb-Liniger model?

Interaction strength and dimensionality

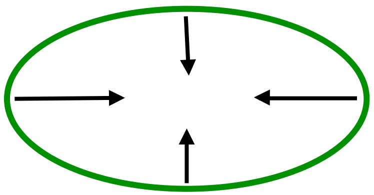


Expect 1D physics when $|\eta| \ll 1$

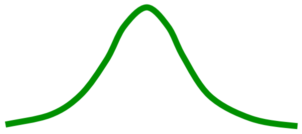
The 1D gas can be dilute even when $\gamma \gg 1$ -> strong correlation

Condensates with Attractive Interactions

- **Collapse** occurs in free space, may be stabilized by trapping potential



- In 1D: no collapse, instead **bright solitons**. The nonlinear Schrödinger equation is *integrable*



- First **observation** of matter-wave bright solitons in 2002 at ENS (Paris) and Rice (Texas) in elongated traps (cigars)
- Soliton trains at Rice pose **riddles**

Quantum description of attractive bosons in 1D

- Exact solutions by J. B. McGuire (1964) for 1D bosons with attractive delta interaction
  - There is exactly one bound state for N particles. This is the ground state
  - All other solutions of N particles are scattering states. The scattering phase shifts can be determined.
- Quantum solitons as superpositions of McGuire bound states (Lai, Haus 1989)
  - Density profile and energies of GPE solitons compares very well with exact solutions
- Phase space/field theory treatment of quantum solitons by Drummond/Carter (1987)
  - Predicts squeezing in the number/phase uncertainty

Ground state for *N* attractive bosons in 1D box (Lai, Haus 1989)

Quantum mechanics (Mc Guire 1964)

- Bound state (cluster) of *N* particles
- Non-degenerate
- CoM delocalised quantum particle

GP mean field theory

- $\phi(x) = \mathrm{sech}(x)$ or $\mathrm{cn}(x|m)$
- Highly degenerate (position of soliton)
- CoM localised classical particle

$$g^2(x - x') = \langle \psi^\dagger(x)\psi^\dagger(x')\psi(x')\psi(x)\rangle \approx \mathrm{sech}^4(x - x')$$

Reality is actually a bit more complicated but in essence the g2 function is bell-shaped in both theories. For a detailed comparison see Kärtner and Haus PRA 48, 2361 (1993).

nature 09 May 2002

letters to nature

Nature **417**, 150 – 153 (2002); doi:10.1038/nature747
Nature AOP, published online 1 May 2002

Formation and propagation of matter–wave soliton trains

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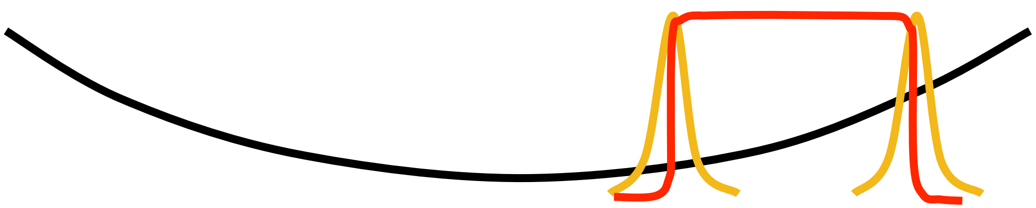
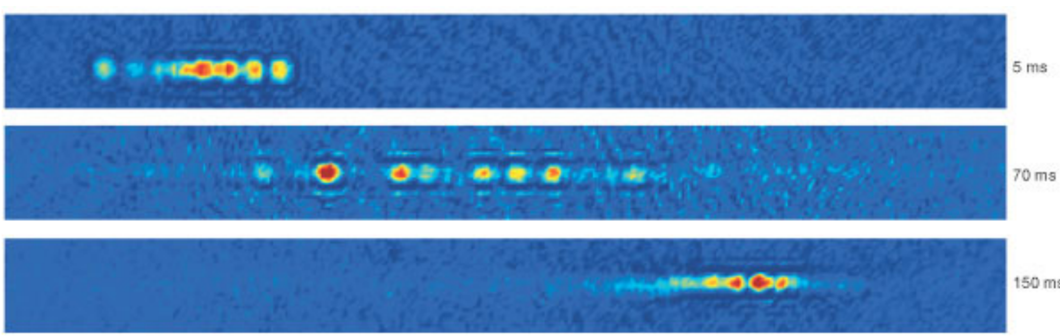
initial density profile





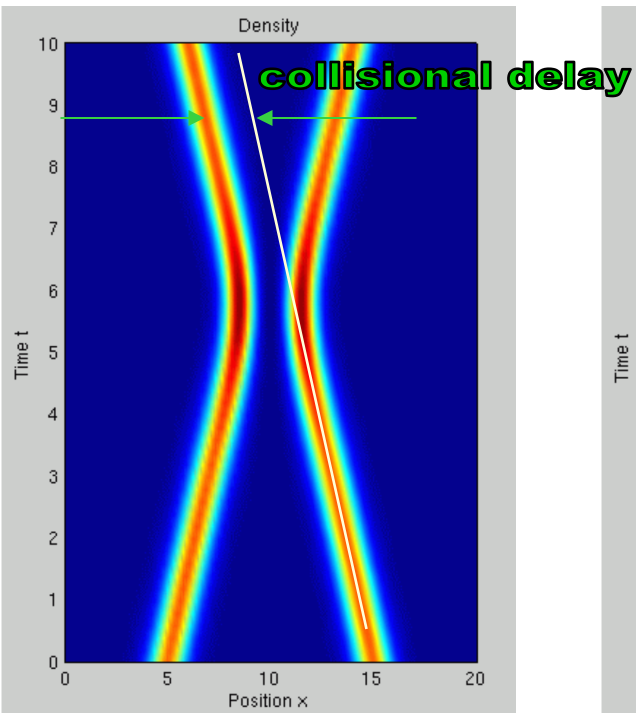
- interactions are switched to attractive, end caps removed
- initial "rectangular" density profile breaks up into train of 4 to 7 solitons
- 90% of atoms are lost
- soliton dynamics shows **repulsive soliton-soliton interactions**

Bright soliton interactions (NLS)

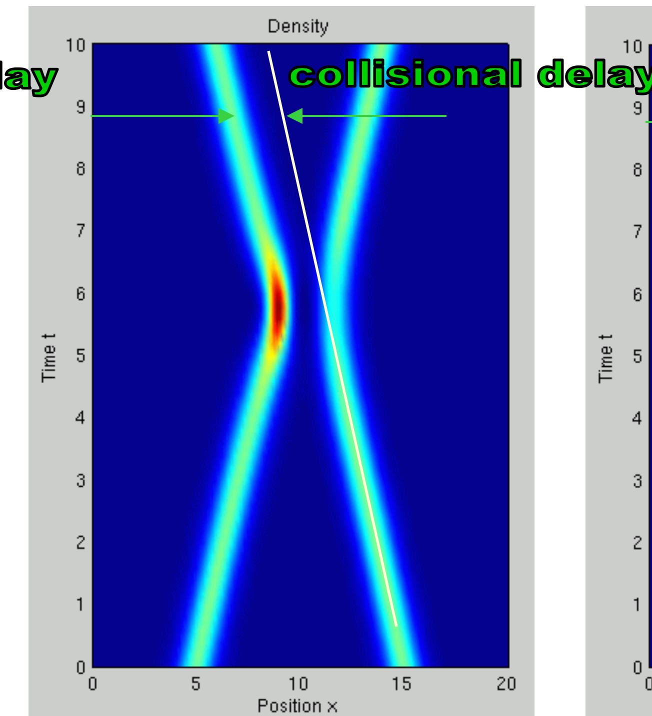
Dynamics of classical particles with short range interaction that depends on the relative phase (J. P. Gordon 1983)

repulsive

$\Delta\phi = \pi$

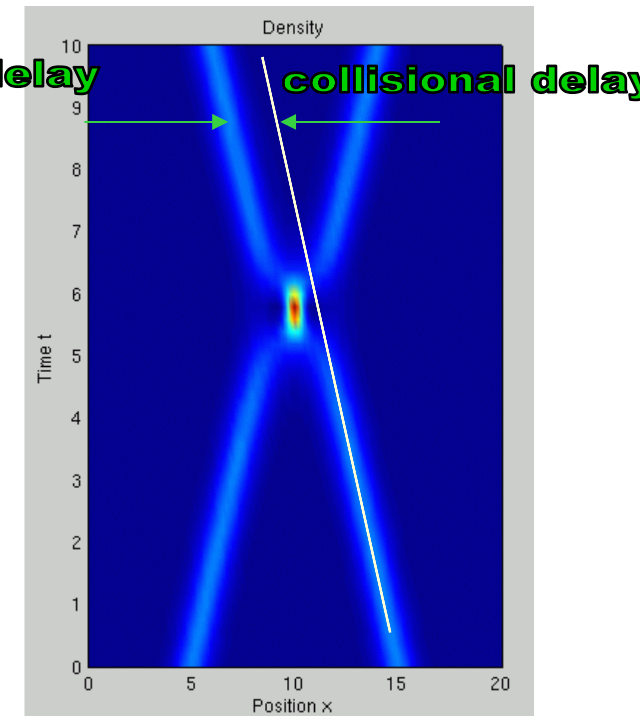


$\Delta\phi = \pi/2$



attractive

$\Delta\phi = 0$



Collisional delay but no mass exchange during collision!

The relative phase of two solitons

- Gross-Pitaevskii (NLS):
  - Always well defined, changes deterministically with time
- Phase-space, field theory approaches:
  - Phase fluctuations occur stochastically due to quantum and/or thermal fluctuations
- Two different number states solitons (this is a fragmented condensate):
  - There is no relative phase. Evolution is deterministic
    - Variational two mode theory seems to predict repulsion of solitons
    - Bethe-ansatz, exact solutions predict ???

Repulsively interacting bosons in 1 dimension: Bosons play fermions

The Lieb-Liniger model and
the Tonks-Girardeau gas

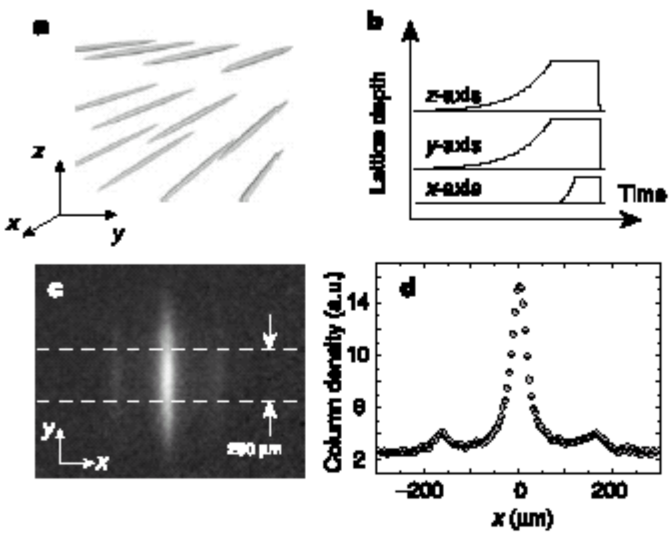
Tonks-gas – Experiments

letters to nature

Tonks–Girardeau gas of ultracold atoms in an optical lattice

Belén Paredes[1], Artur Widera[1,2,3], Valentin Murg[1], Olaf Mandel[1,2,3], Simon Fölling[1,2,3], Ignacio Cirac[1], Gora V. Shlyapnikov[4], Theodor W. Hänsch[1,2] & Immanuel Bloch[1,2,3]

MPQ Garching



up to $\gamma_{eff} \sim 200$

other experiments:

T. Esslinger (Zürich)

W. Phillips (NIST)

D. Weiss (PSU), $\gamma \sim 5.5$

R. Grimm (Innsbruck): confinement induced resonance!

$$\gamma \approx \frac{\text{interaction energy}}{\text{kinetic energy}}$$

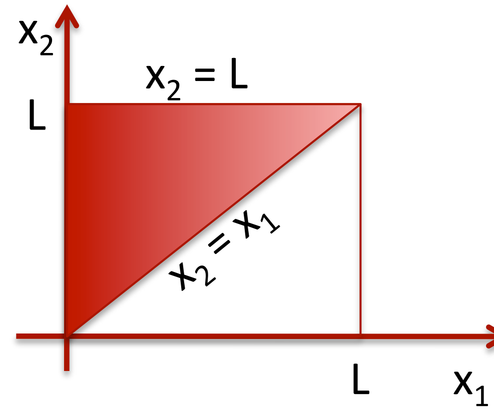
$$\gamma \simeq \frac{m}{\hbar}\frac{\omega_\rho}{n_{1D}} a_{3D}$$

1D Bose Gas – Lieb-Liniger model

$$H = \sum_i \frac{p_i^2}{2m} + 2g_{1\mathrm{D}} \sum_{i<j} \delta(x_i - x_j)$$

- 1D Bosons with repulsive δ interactions
- Ground- and excited-state wavefunctions exactly known from Bethe ansatz [Lieb, Liniger (1963)]
- Interaction parameter $\gamma = \frac{m}{\hbar^2} \frac{g_{1\mathrm{D}}}{n}$
- For $\gamma \to \infty$, problem is mapped exactly to free Fermi gas (Tonks-Girardeau gas) [Girardeau (1960)]
- Ring geometry provides periodic boundary conditions

Lieb-Liniger model: wave function



Consider $0 \le x_1 \le x_2 \cdots \le x_N \le L$

- Inside: $$-\frac{\hbar^2}{2m}\sum_i \frac{\partial^2}{\partial x_i^2}\psi = E\psi$$
- Boundary conditions are provided by
  - Interactions
  - Periodicity in the box
- Bethe ansatz:

$$\psi(x_1,\ldots,x_N) = \sum_P a(P) P \exp(i\sum_{j=1}^{N} k_j x_j)$$

P is a permutation of the set $\{k\} = k_1, k_2, \ldots, k_N$

- Just one quasimomentum per particle (!)
- Model is integrable, check Yang-Baxter equation

Bose-Fermi mapping

"In 1D, there is no distinction between Bosons and Fermions"

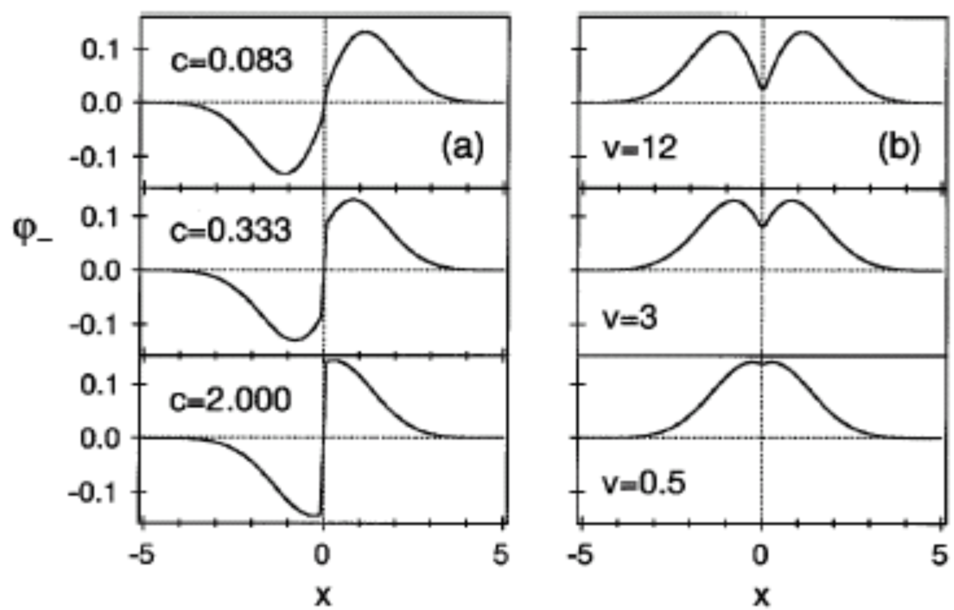
Strong repulsive interactions for bosons have the same effect as the Pauli exlusion principle for fermions.

The 1D Bose gas maps one-to-one to a gas of spinless fermions

$$\phi^B = |\phi^F|$$

Bosons with strong but finite interactions map to spinless fermions with weak short-range interactions

Fermions Bosons



Cheon and Shigehara, 1999

Girardeau, 1960

Pseudopotential in the Fermionic picture

Sen's pseudopotential generates correct energy levels to first order in $1/\gamma$

$$V(x_1, x_2) = -\frac{2\hbar^2}{mc}\delta''(x_1 - x_2)$$ [D. Sen 1999]

generalization for arbitrary γ:

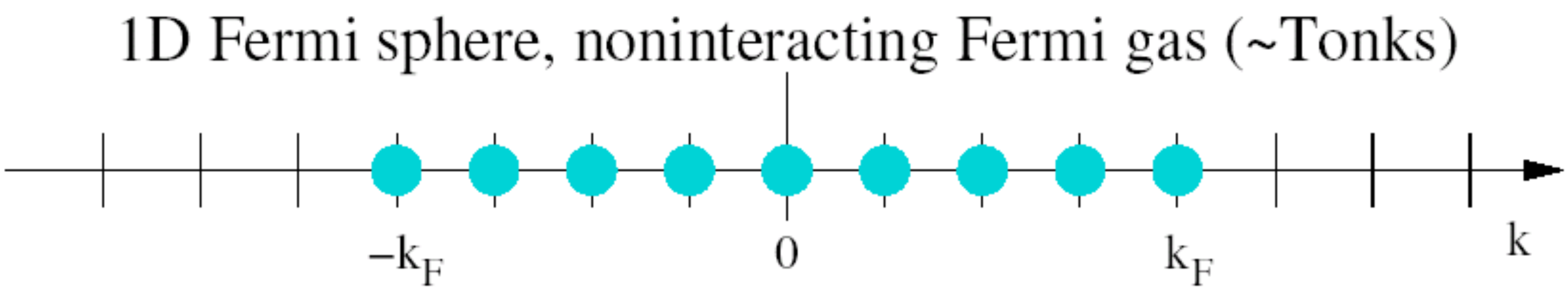
$$V(x_1, x_2, x_2', x_1') = -\frac{4\hbar^2}{mc}\delta\left(\frac{x_1 + x_2 - x_2' - x_1'}{2}\right)\delta'(x_1 - x_2)\delta'(x_1' - x_2')$$

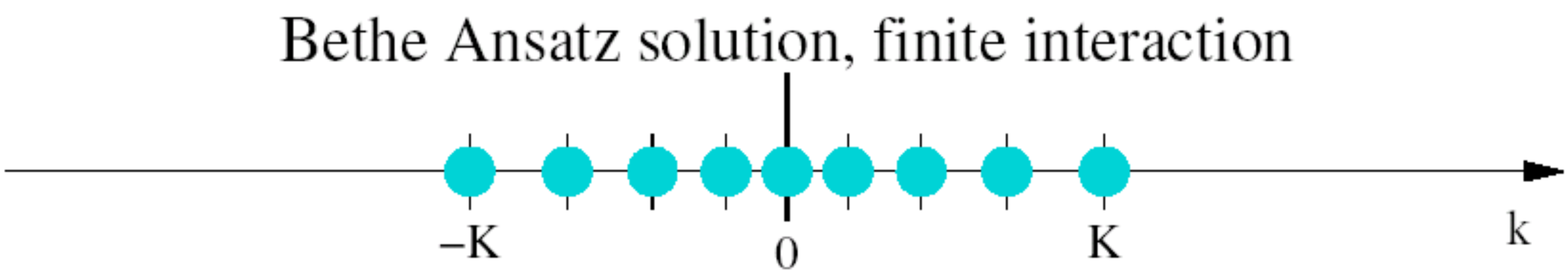
Granger and Blume [2004],
Girardeau and Olshanii [2004],
Brand and Cherny [2005]

This can be used to apply common methods of fermionic many-body theory, e.g.

- Hartree-Fock
- diagrammatic many-body perturbation theory
- Random-phase approximation

The nature of Bethe-ansatz solutions: Quasi-momenta and Fermi sphere





Total energy: $E = \frac{\hbar^2}{2m} \sum_k k^2$

Total momentum: $P = \hbar \sum_k k$

Lieb-Liniger ground states

The quasi momentum distribution in the ground state is deformed from the simple Fermi-sphere picture at finite (weaker) interactions

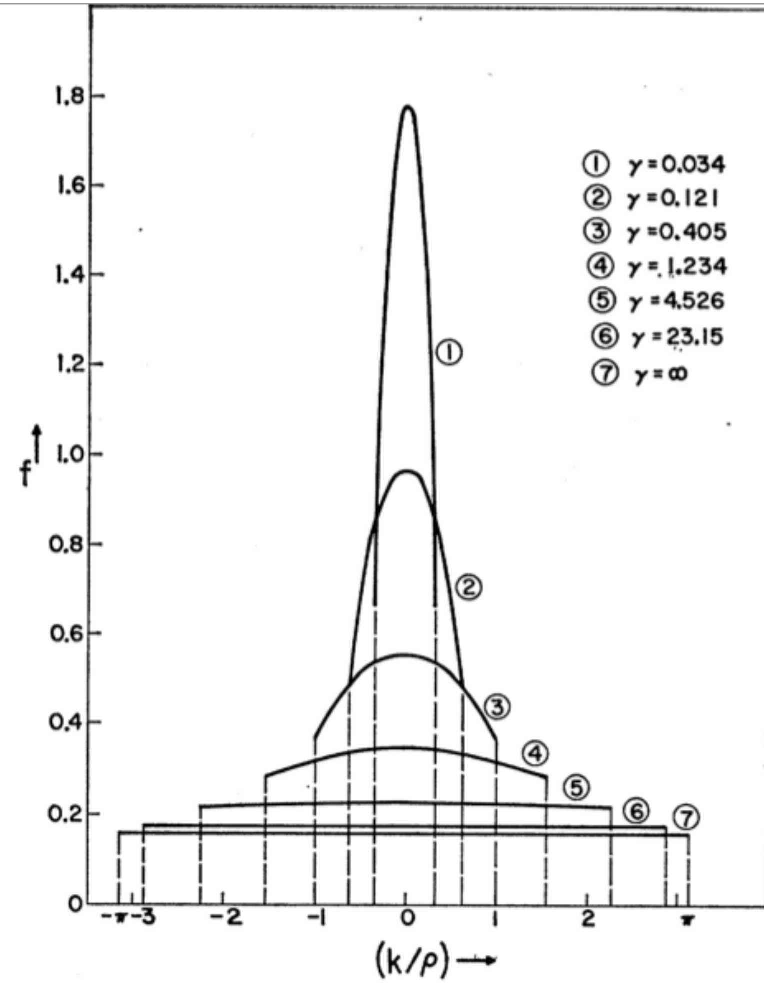


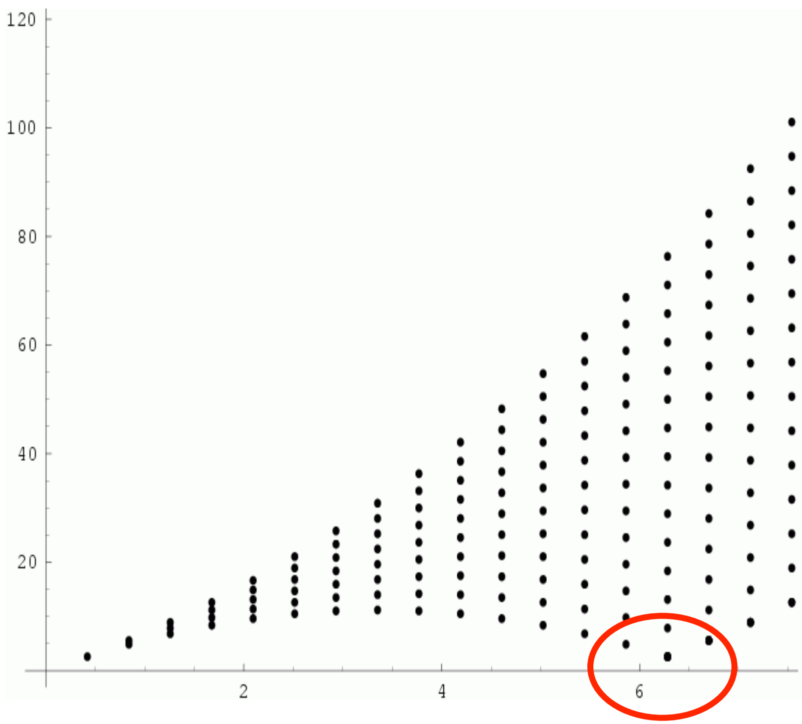
FIG. 2. The distribution function of "quasi-momenta" in the ground state for various values of $\gamma=c/\rho$. The vertical dashed lines are the cutoff momenta K (cf. Fig. 1). When $\gamma=\infty$, $f=(2\pi)^{-1}$. For all γ, $\int_{-K}^{K} f(k)dk=\rho$.

Lieb, Liniger, 1963

Excitation spectrum for the Lieb-Liniger model

$\gamma = \infty$

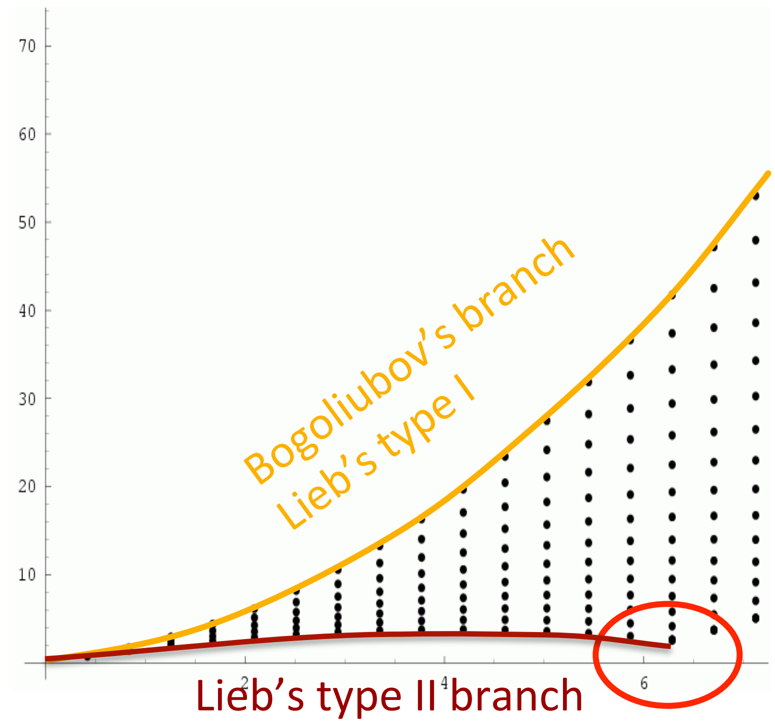
$\sim\omega\pi^2/\varepsilon_F$



momentum $q\pi/k_F$

$\gamma=1$

$\sim\omega\pi^2/\varepsilon_F$



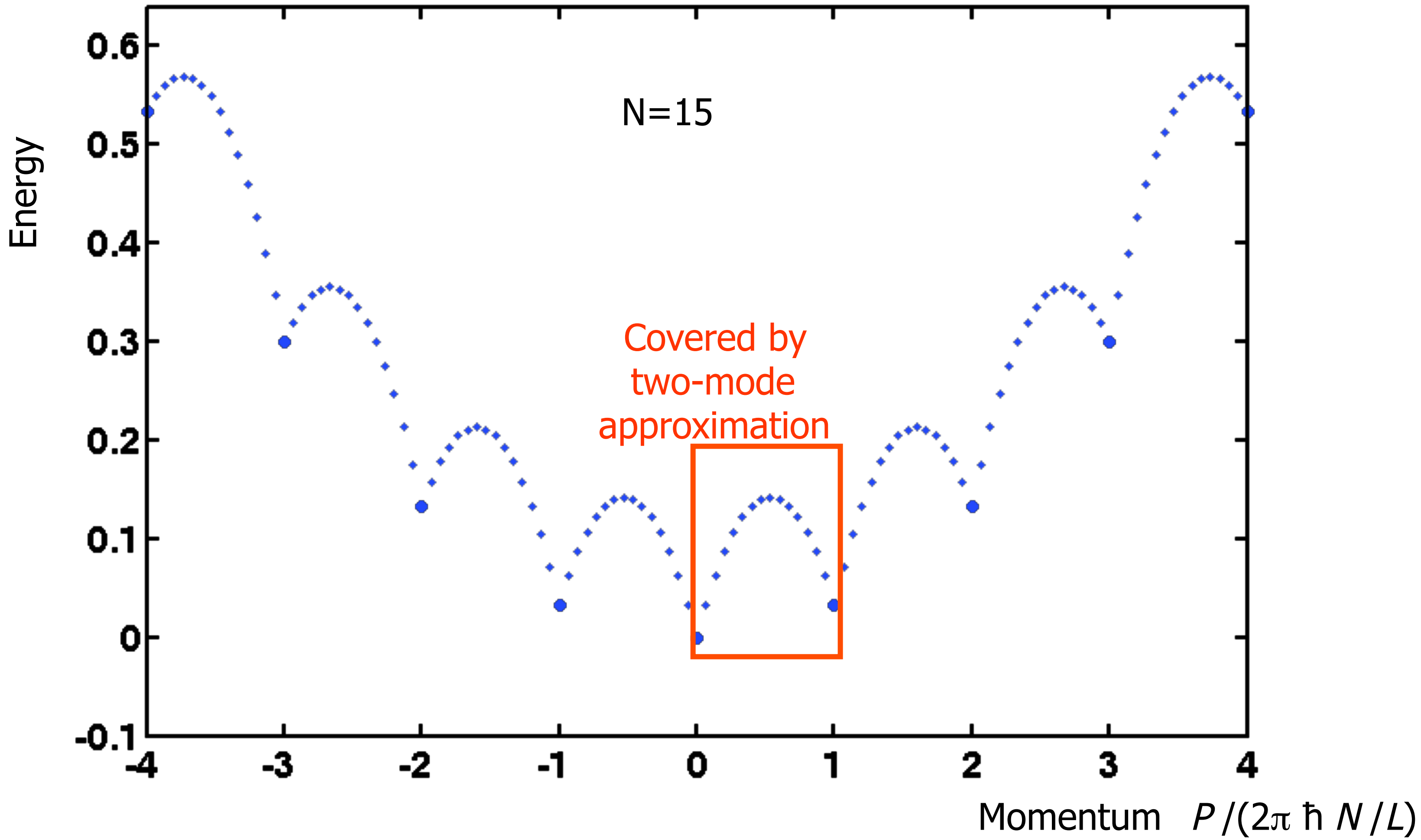
momentum $q\pi/k_F$

umklapp excitation

$k_F=\pi n_{1D}$; $\varepsilon_F=\sim^2 k_F^2/(2m)$

Type II excitations can be identified with dark solitons!

Low-lying excitation spectrum (yrast states)



Dark soliton dispersion relation (for GPE solution)

Energy

$$E = W - W_0$$

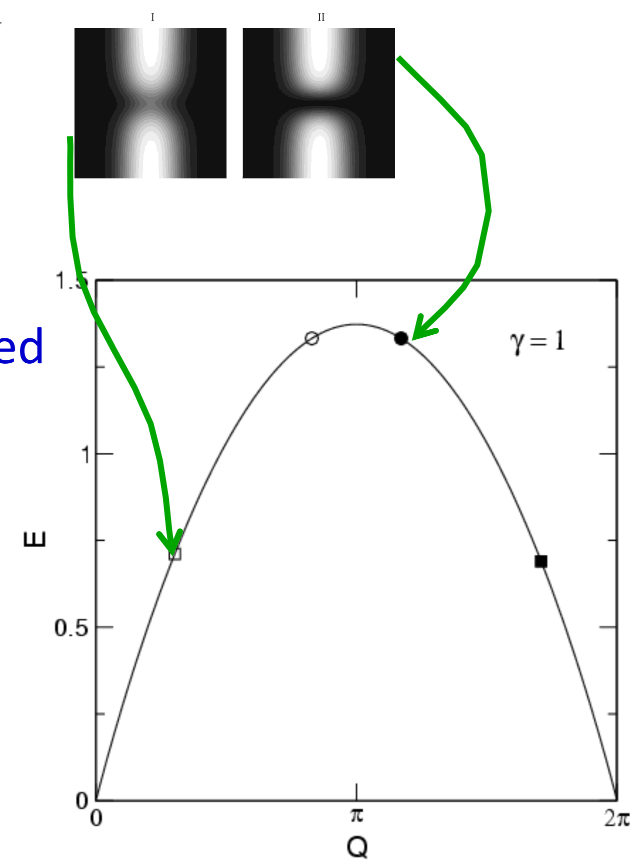
$$W = \frac{1}{2}\int \left[|\nabla\Psi|^2 + \rho^2|\Psi|^2 + 4\pi\gamma|\Psi|^4 - 2\mu|\Psi|^2 \right] dV$$

Impulse (canonical momentum)

$$Q = \int (n - n_0)\frac{\partial\phi}{\partial z} dV$$

The velocity has an upper bound in the speed of sound v_s.

$$v = \frac{dE}{dQ} < v_s$$



Dispersion relation for dark solitons

The dark soliton dispersion (in the right units) asymptotically matches the Lieb type II dispersion relation for large densities. Ishikawa, Takayama JPSJ (1980)

Two-mode model

Restrict particles to zero or single unit of momentum.
This is valid for *N* particles in the limit of small interactions.

Dark solitons

- Bose-Einstein condensed

$$\left(\alpha a_0^\dagger + \beta a_1^\dagger\right)^N |\text{vac}\rangle$$

Yrast states

- Fragmented condensate

$$\left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\text{vac}\rangle$$

How could these two be related?

Which one is correct?

Two-mode model

Restrict particles to zero or single unit of momentum.
This is valid for *N* particles in the limit of small interactions.

Dark solitons

- Bose-Einstein condensed

$$\left(\alpha a_0^\dagger + \beta a_1^\dagger\right)^N |\mathrm{vac}\rangle$$

$$= \sum_p \binom{N}{p} \gamma_p \left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\mathrm{vac}\rangle$$

Yrast states

- Fragmented condensate

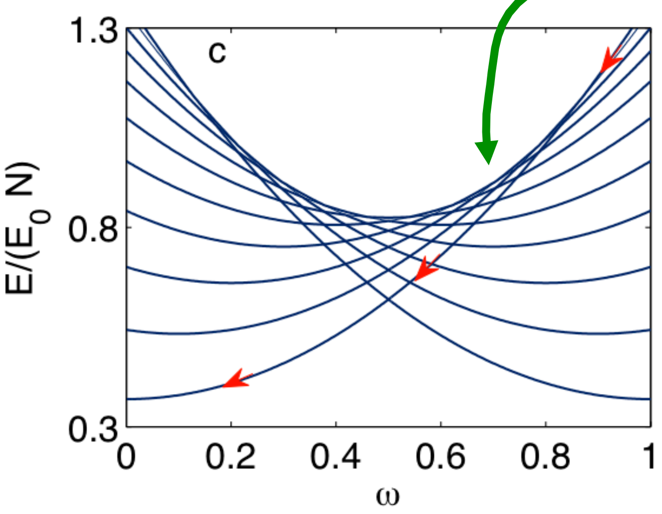
$$\left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\mathrm{vac}\rangle$$

$$\rightarrow \sum_p c_p \left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\mathrm{vac}\rangle$$

Becomes a multiple superposition due to the symmetry breaking potential

Energy level diagram

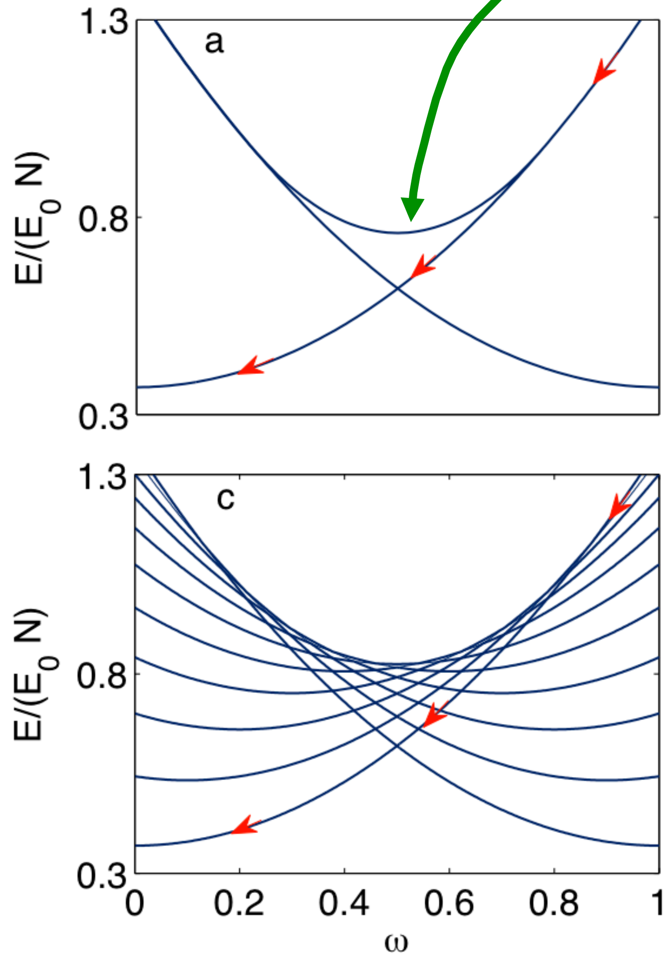
Caustic area
(many lines intersect)



Quantum states
(two mode approximation)

Energy level diagram

Dark soliton



Mean field (GP) stationary states; Plane waves (ring currents) and dark soliton

Quantum states
(two mode approximation)

Kanamoto et al. identified a metastable QPT through yrast states.
Can it be achieved?

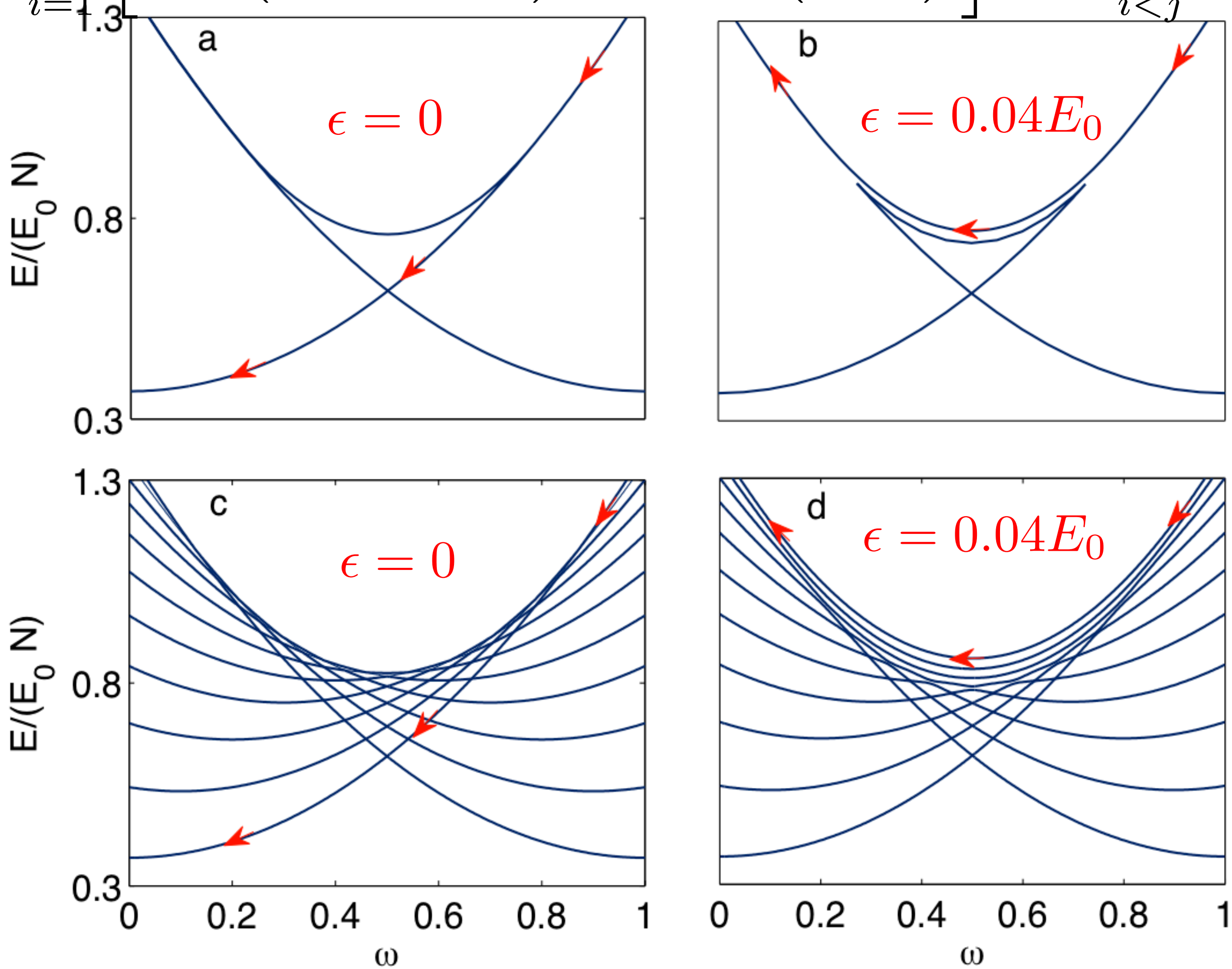
Kanamoto, Carr, Ueda, PRL (2008), PRA (2009, 2010)

Add symmetry breaking potential

$$H = \sum_{i=1}^{N} \left[\frac{\hbar^2}{2m} \left(-i \frac{\partial}{\partial x_i} - \frac{\omega}{L} \right)^2 + \epsilon \cos\left(\frac{2\pi x_i}{L} \right) \right] + g_1 \sum_{i<j}^{N} \delta(x_i - x_j)$$

Mean field (GP)

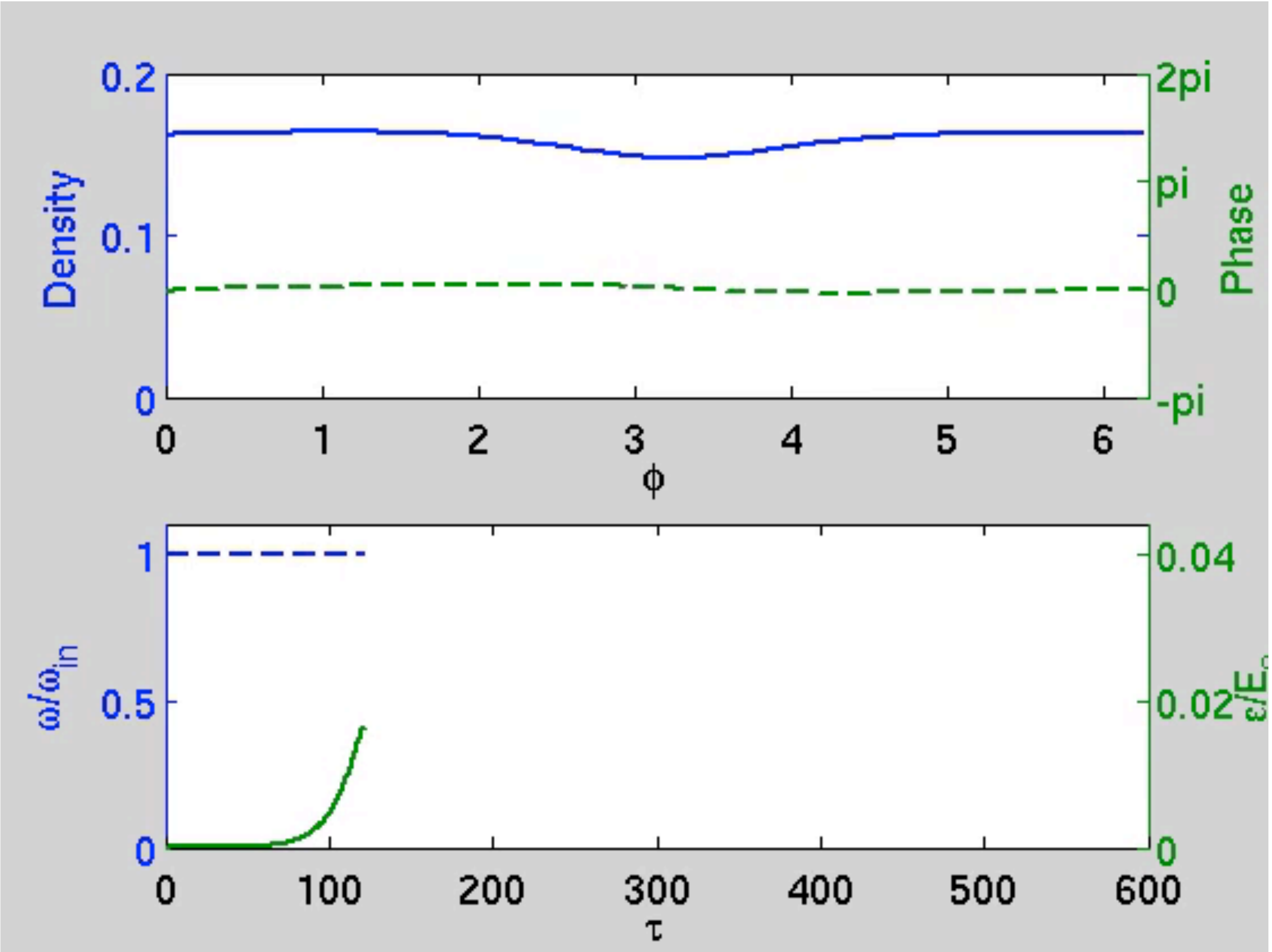
Quantum (two-mode)



The level splitting creates an adiabatic passage

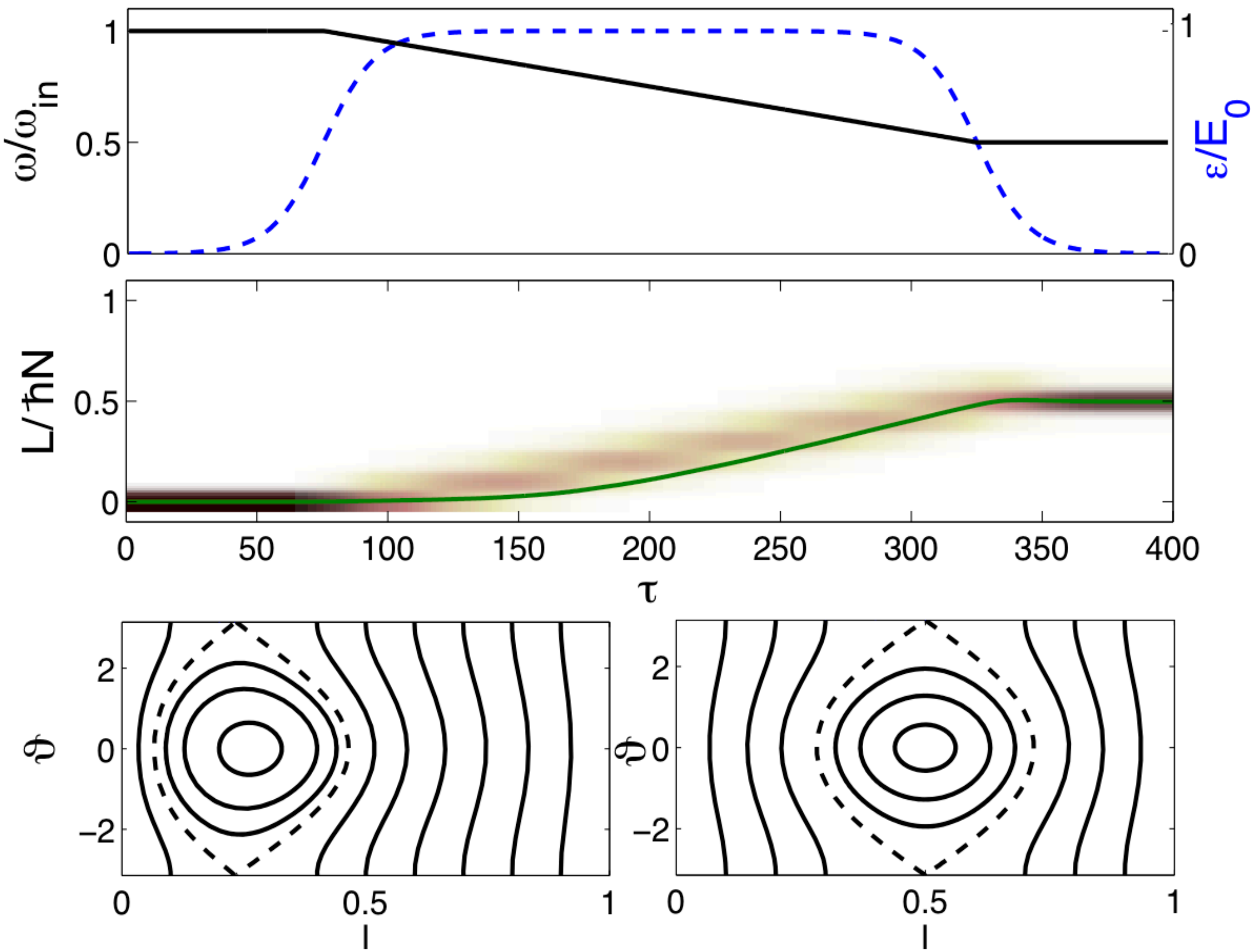
Fialko, Delattre, Brand, Kolovsky, PRL (2012)

Simulation of adiabatic passage (GP)



Final state: dark soliton

Time dependent simulation of the adiabatic passage



Final state: dark soliton

Symmetry is finally broken.
Can it be restored?

$$\rightarrow \sum_p c_p \left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\text{vac}\rangle$$

- Removing the symmetry breaking potential *adiabatically* should restore symmetry…

$$\left(a_0^\dagger\right)^{N-p} \left(a_1^\dagger\right)^p |\text{vac}\rangle$$

… however, the time scale diverges with *N*.
For large particle number, the symmetry remains broken?

"More is different"

Wrap up

- 1D physics is different from 3D and very rich
- 1D quantum gases are experimentally accessible
- Exactly solvable models give valuable insights (and exact results)
- Even though it is not *a priori* clear that mean field theory can be trusted, it can give useful predictions and insights (if treated with a grain of salt)

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