Physics of ultracold Bose gases in one-dimension and solitons

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One dimension is different!

- To be covered in these lectures:
  - Introduction to solitons
  - Absence of true Bose-Einstein condensation
  - Strongly-correlated many-body physics with a dilute gas
  - Attractive bosons and quantum bright solitons
  - Bosons play fermions: Lieb-Liniger model
  - Superfluid or not superfluid (or maybe both?)
  - Where are solitons in the Lieb-Liniger model?

One dimension is different: There is no BEC.
Does that mean that Gross-Pitaevskii theory is meaningless?

No BEC here?

What is BEC?

What is Bose-Einstein condensation?

- Defined through scaling property of single-particle density matrix (spdm) :

$$g(x,x') = \langle \psi^\dagger(x)\psi(x')\rangle = N\int dx_2 \ldots dx_N \Psi^*(x,x2,\ldots,x_N)\Psi(x',x2,\ldots,x_N)$$

- For an infinite system we expect off-diagonal long range order (ODLRO):

$$\lim_{|x-x'|\to\infty} g(x,x') = n_c > 0$$

- For a finite system we can look at *natural orbitals*:

$$g(x,x') = \sum_k n_k \phi_k^*(x)\phi_k(x') \qquad \int \phi_k^*(x)\phi_l(x) = \delta_{kl} \qquad \sum_k n_k = N$$

- In the thermodynamic limit we want

$$\lim_{N\to\infty} \frac{n_0}{N} = f_c > 0$$

This is BEC!

$$n_c = f_c \frac{N}{V}$$

Thermodynamic limit

- For the thermodynamic limit we assume a box with linear size *L* (and periodic boundaries or ring)

$$N \to \infty, L \to \infty$$

- 3D: $n_3 = \frac{N}{L^3} = \text{const.}$ BEC phase transition (finite T and interaction)
- 2D: $n_2 = \frac{N}{L^2} = \text{const.}$ Berezinski-Kosterlitz-Thouless PT
- 1D: $n_1 = \frac{N}{L} = \text{const.}$ no PT (Yang-Yang)
- Absence of BEC phase transition for d<3 follows from Mermin-Wagner theorem (c.f. Hohenberg, Coleman)

1D Bose gas

- Homogeneous gas (e.g. large-radius ring trap):
  - No phase transition and no ODLRO
  - Fluctuations of phase are large (diverge for infinite system)
  - Finite T: exponential decay of spdm
  - Zero T: algebraic decay of spdm
- Harmonically trapped 1D Bose gas:
  - BEC is possible (Ketterle, van Druten)
  - Length scale for phase fluctuations should be compared to Thomas-Fermi radius of gas

1D Bose gas in harmonic trap

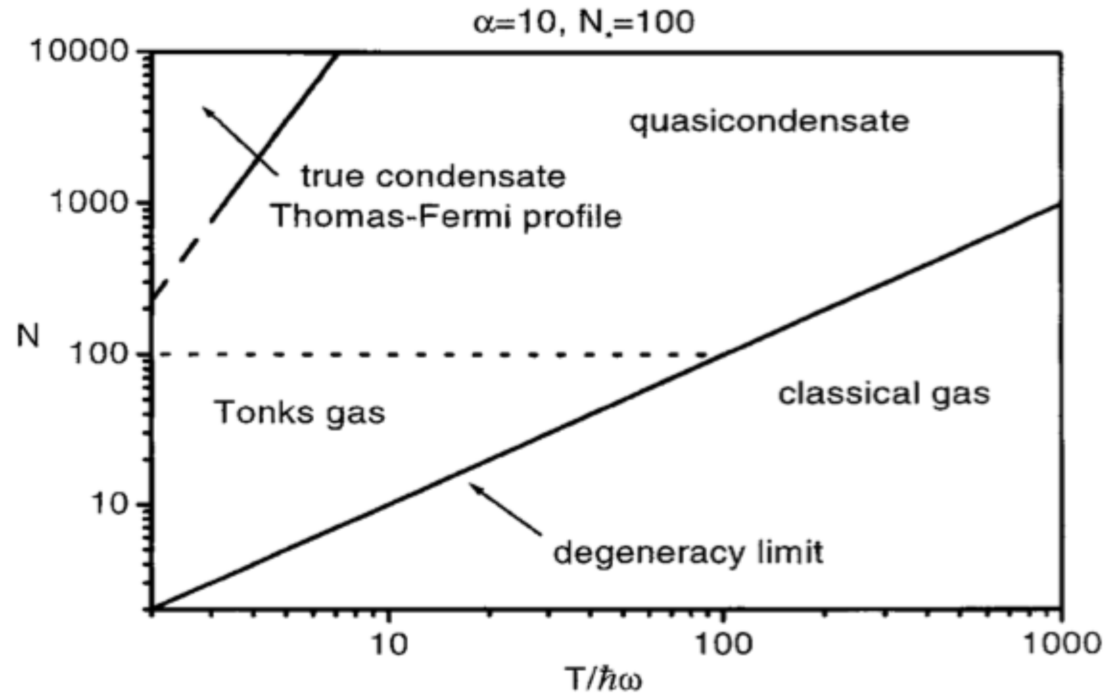
- Degeneracy temperature $$T_d \approx \frac{N\hbar\omega}{k_B}$$

- Phase fluctuations dominate in the quasicondensate regime but freeze out at

$$T_{ph} = \frac{T_d \hbar\omega}{\mu}$$

- Crossover to BEC at

$$T_c \approx \frac{N\hbar\omega}{k_B \ln 2N}$$



Petrov (2000)

Phase fluctuating condensate?

- Bogoliubov's trick:

$$\hat{\psi}(x) = \phi(x) + \delta\hat{\psi}(x)$$

This obviously works if BEC is present (3D).

However, it is sufficient to have small density fluctuations (works in 1D without BEC):

$$\hat{\rho}(x) = \hat{\psi}^\dagger(x)\hat{\psi}(x) \approx \rho_0 + \delta\hat{\rho}(x)$$

The (fluctuating) phase is then "defined" by

$$\hat{\psi}(x) = \sqrt{\hat{\rho}}e^{\hat{\theta}}$$

Y. Castin, **Simple theoretical tools for low dimension Bose gases**, J. Phys. IV France, 116, 89 (2004) arXiv:0407118

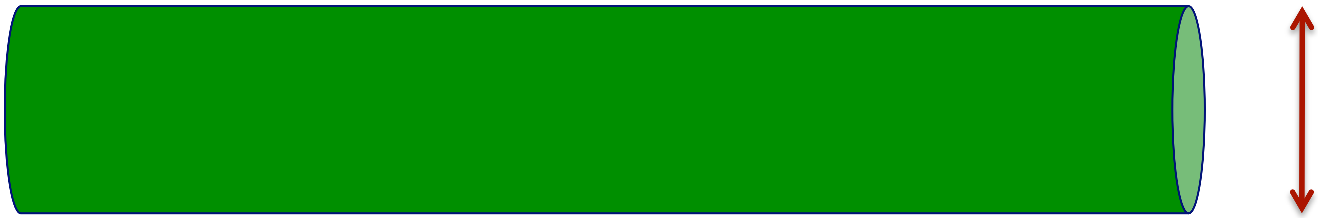
V. N. Popov, **Functional Integrals in Quantum Field Theory and Statistical Physics**, (Reidel, Dordrecht, 1983).

Strongly correlated and yet dilute?

The dimensional crossover

From 3D to 1D

- Consider cylindrical trap $V_{\text{trap}} = \frac{1}{2} m \omega_\perp^2 (x^2 + y^2)$

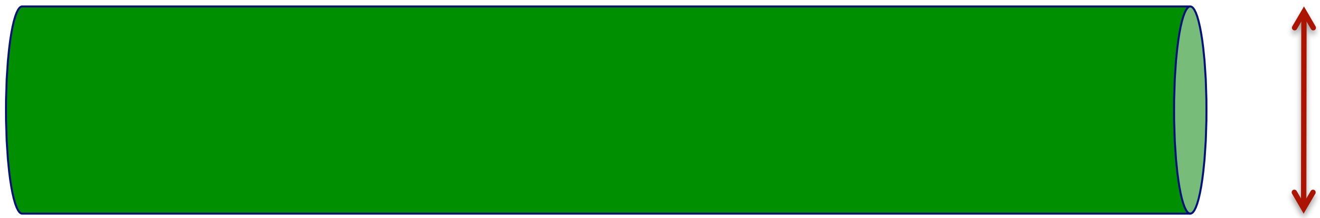


$$l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

- 3D coupling strength: $$g_3 = \frac{4\pi\hbar^2 a_s}{m}$$
- 1D coupling strength: $$g_1 = \frac{2\hbar^2 a_s}{m l_\perp^2} = \frac{g_3}{2\pi l_\perp^2}$$

  [more accurately $$g_1 = \frac{2\hbar^2 a_s}{m l_\perp^2}(1 - C a_s / l_\perp)^{-1}$$
  (Olshanii 1998), leads to confinement-induced resonance!]
- Healing length: $$l_c = \frac{\hbar}{\sqrt{m n_3 g_3}} \approx \frac{\hbar}{\sqrt{m n_1 g_1}}$$

Dimensionless interaction strength



$$l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

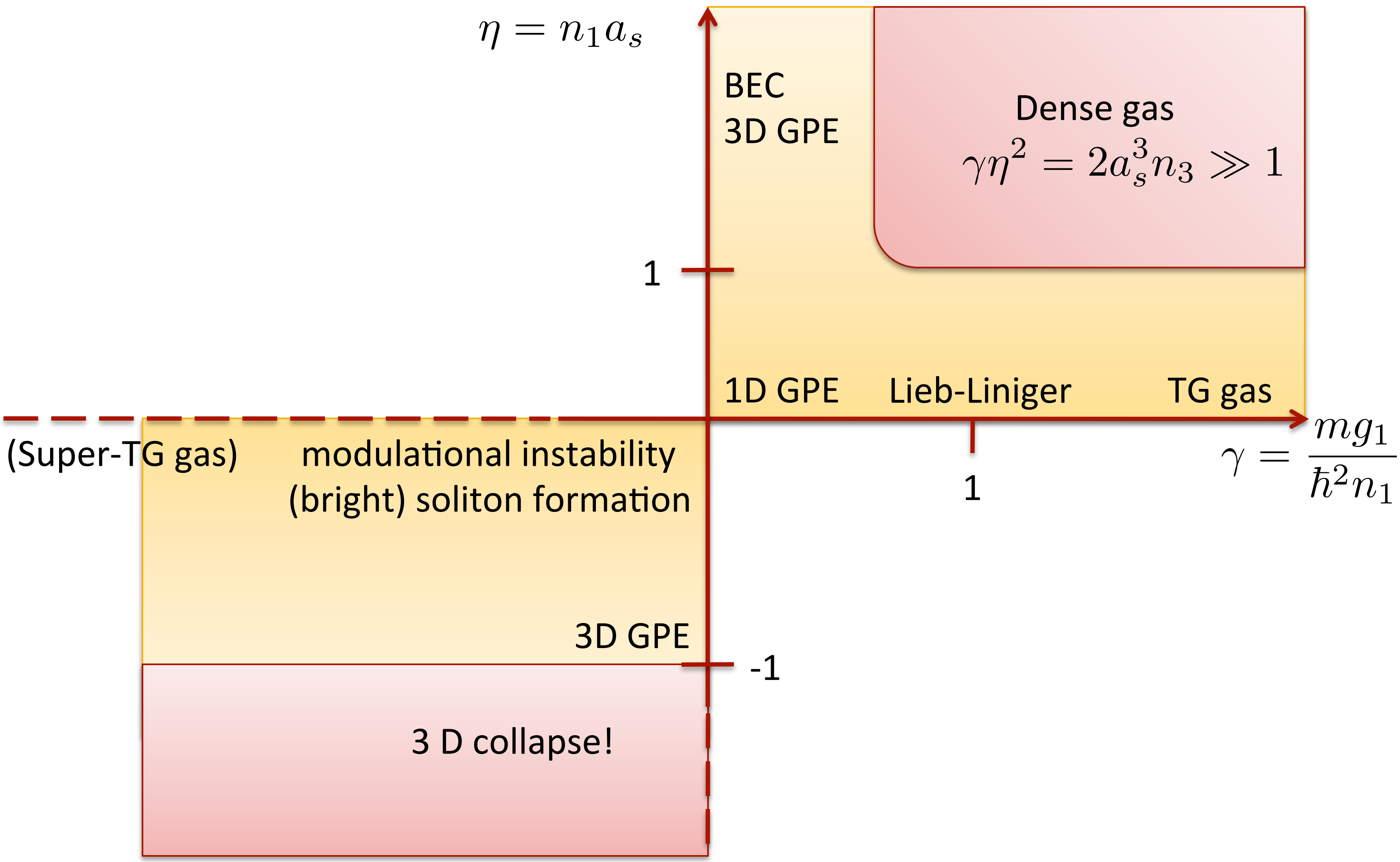
- Lieb-Liniger parameter: $$\gamma = \frac{m g_1}{\hbar^2 n_1} = \frac{1}{(n_1 l_c)^2}$$

  compares mean distance between particles and healing length

- Komineas-Papanicolao parameter: $$\eta = n_1 a_s = \frac{1}{2}\left(\frac{R_\perp}{l_c}\right)^2 \approx \frac{\mu}{\hbar\omega_\perp}$$

  compares healing length with transverse Thomas-Fermi radius (Komineas 2002)

Interaction strength and dimensionality

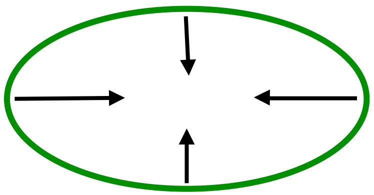


Expect 1D physics when $|\eta| \ll 1$

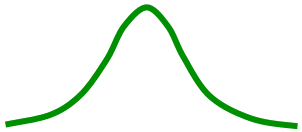
The 1D gas can be dilute even when $\gamma \gg 1$ -> strong correlation

Condensates with Attractive Interactions

- **Collapse** occurs in free space, may be stabilized by trapping potential



- In 1D: no collapse, instead **bright solitons**. The nonlinear Schrödinger equation is *integrable*



- First **observation** of matter-wave bright solitons in 2002 at ENS (Paris) and Rice (Texas) in elongated traps (cigars)
- Soliton trains at Rice pose **riddles**

Quantum description of attractive bosons in 1D

- Exact solutions by J. B. McGuire (1964) for 1D bosons with attractive delta interaction
  - There is exactly one bound state for N particles. This is the ground state
  - All other solutions of N particles are scattering states. The scattering phase shifts can be determined.
- Quantum solitons as superpositions of McGuire bound states (Lai, Haus 1989)
  - Density profile and energies of GPE solitons compares very well with exact solutions
- Phase space/field theory treatment of quantum solitons by Drummond/Carter (1987)
  - Predicts squeezing in the number/phase uncertainty

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letters to nature

Nature **417**, 150 – 153 (2002); doi:10.1038/nature747
Nature AOP, published online 1 May 2002

Formation and propagation of matter–wave soliton trains

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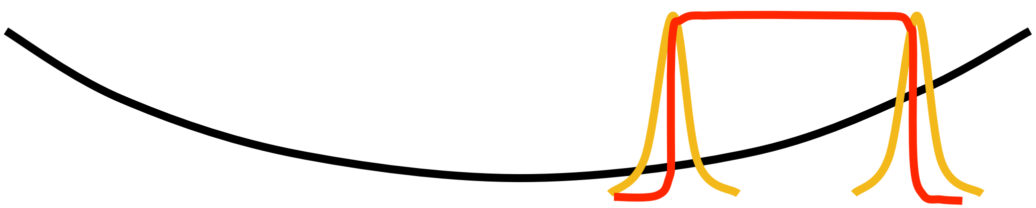
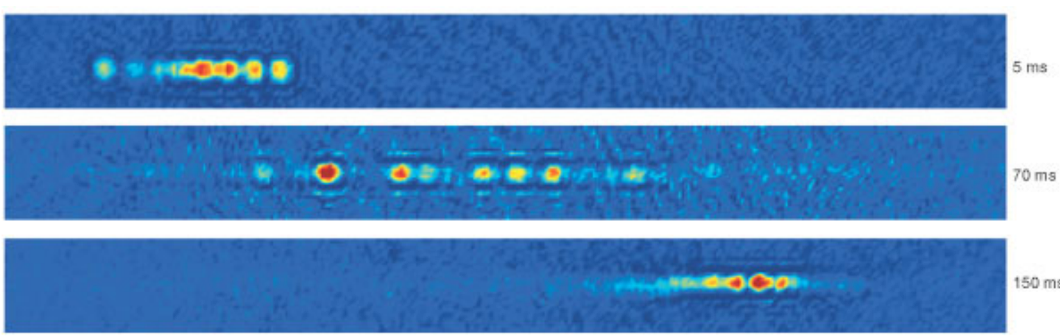
initial density profile



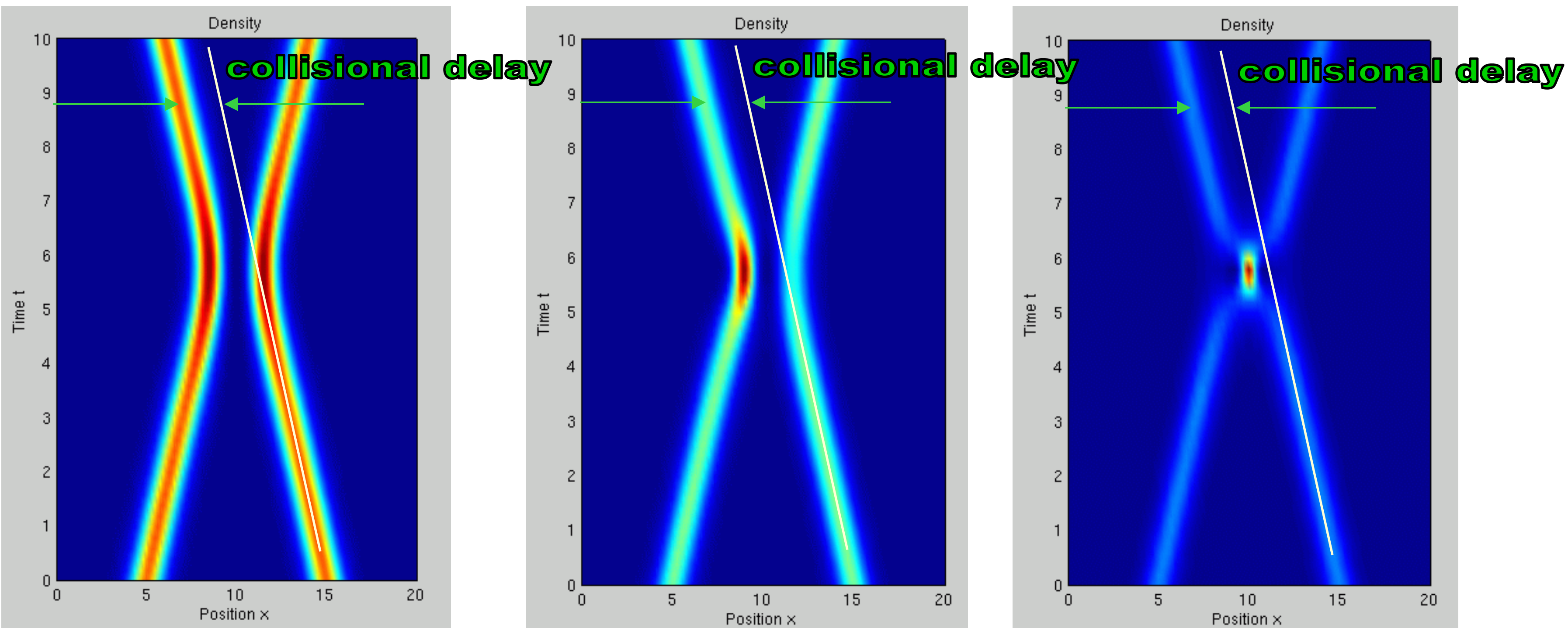


- interactions are switched to attractive, end caps removed
- initial "rectangular" density profile breaks up into train of 4 to 7 solitons
- 90% of atoms are lost
- soliton dynamics shows **repulsive soliton-soliton interactions**

Bright soliton interactions (NLS)

Dynamics of classical particles with short range interaction that depends on the relative phase (J. P. Gordon 1983)

repulsive $\Delta\phi = \pi$ | $\Delta\phi = \pi/2$ | attractive $\Delta\phi = 0$



Collisional delay but no mass exchange during collision!

The relative phase of two solitons

- Gross-Pitaevskii (NLS):
  - Always well defined, changes deterministically with time
- Phase-space, field theory approaches:
  - Phase fluctuations occur stochastically due to quantum and/or thermal fluctuations
- Two different number states solitons (this is a fragmented condensate):
  - There is no relative phase. Evolution is deterministic
    - Variational two mode theory seems to predict repulsion of solitons
    - Bethe-ansatz, exact solutions predict ???

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