Physics of ultracold Bose gases in one-dimension and solitons

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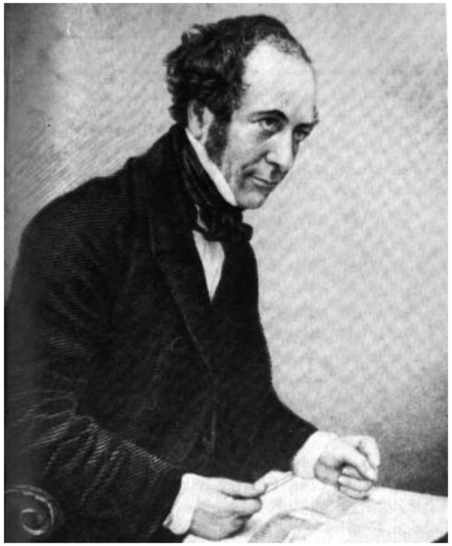
One dimension is different!

- To be covered in these lectures:
  - Introduction to solitons
  - Absence of true Bose-Einstein condensation
  - Strongly-correlated many-body physics with a dilute gas
  - Bosons play fermions: Lieb-Liniger model
  - Superfluid or not superfluid (or maybe both?)
  - Where are solitons in the Lieb-Liniger model?

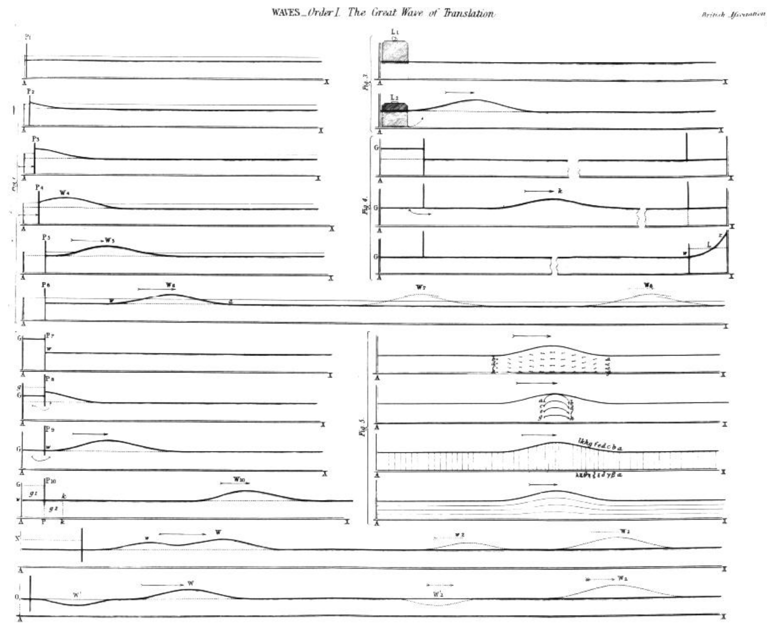
What are solitons?

Solitons

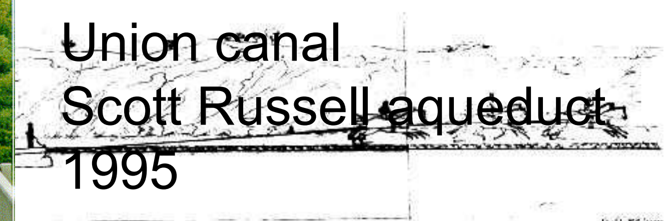
1834



John Scott Russell
(1808 – 1882)
Scottish engineer
witnesses 1834 the
“great wave
of translation “



Explanation:
Korteweg-de Vries equation (1895)

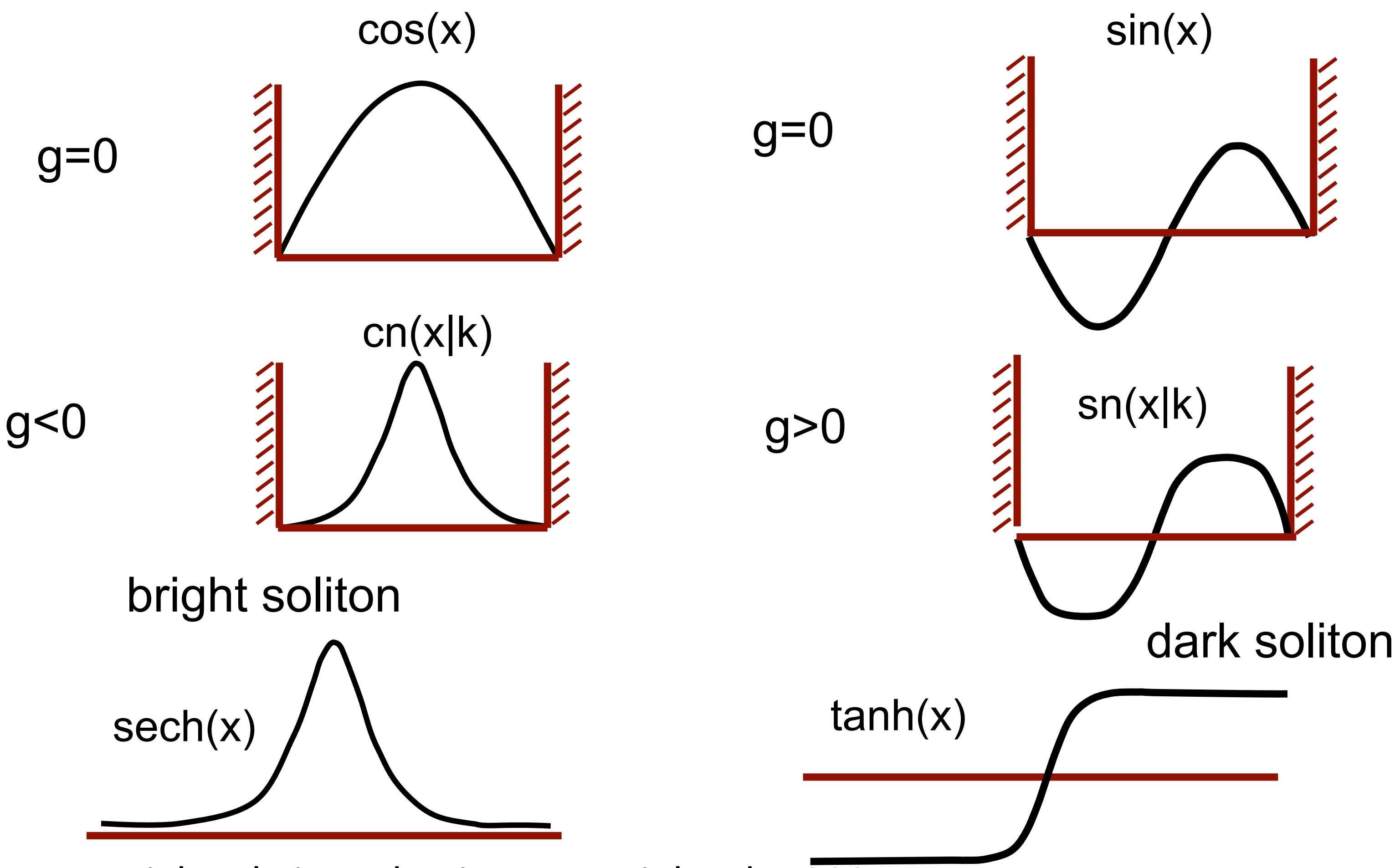


Union canal
Scott Russell aqueduct
1995

ar Edinburgh)

Solitons as stationary solutions of the nonlinear Schrödinger equation

$$i\frac{\partial}{\partial t}u(x,t) = [-\frac{1}{2}\frac{\partial^2}{\partial x^2} + g|u|^2]u(x,t)$$



For a tutorial-style introduction see Reinhardt 1988

Solitons

in the nonlinear Schrödinger equation (NLS)

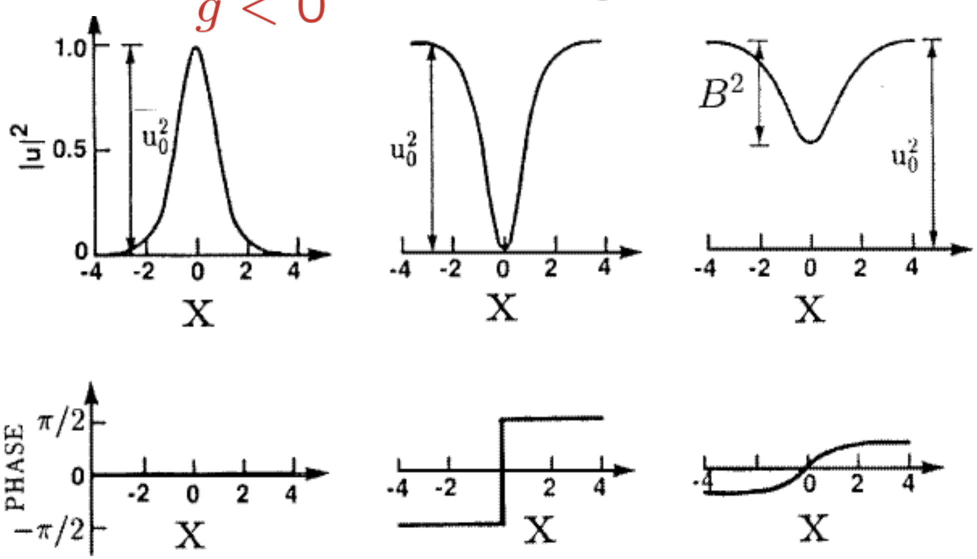
Dispersion

$$i\frac{\partial}{\partial t}u(x,t) = [-\frac{1}{2}\frac{\partial^2}{\partial x^2} + g|u|^2]u(x,t)$$

Nonlinearity

bright soliton $g < 0$

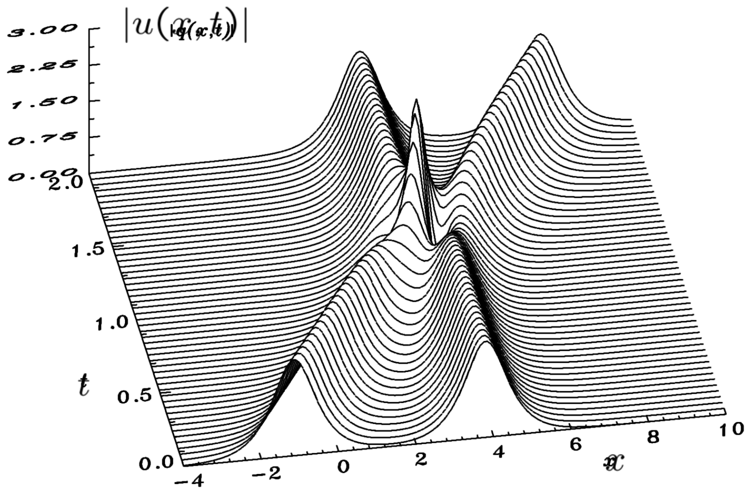
dark solitons $g > 0$



Solitons…

…on the beach of Hawaii

are ***robust***!



Soliton collisions are elastic, show *particle character!*

Bose-Einstein condensation and solitons

Significance of Solitons?

- Optical fibre communications
- Energy and charge transport on molecular chains
- Models for particle theory

Why Solitons in BECs?

- Clean system, low temperatures: ideal realisation of nonlinear Schrödinger equation
- Model for He II, superconductors, etc.
- Potential applications in matter-wave interferometry

Theory: Bose-Einstein Condensate (BEC)

- Bose gas in an external potential

$$i\hbar\frac{\partial}{\partial t}\widehat{\Psi}(\boldsymbol{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{\text{ext}}(\boldsymbol{r},t)+\int\widehat{\Psi}^{\dagger}(\boldsymbol{r}',t)V(\boldsymbol{r}'-\boldsymbol{r})\widehat{\Psi}(\boldsymbol{r}',t)\mathrm{d}\boldsymbol{r}'\right]\widehat{\Psi}(\boldsymbol{r},t)$$

For BECs we may use the classical or mean field (Hartree) approximation:

Interaction becomes a tunable parameter

Gross-Pitaevskii equation

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t)=\left[-\frac{\hbar^2}{2m}\nabla^2+V_{\text{ext}}(\boldsymbol{r},t)+\frac{4\pi a_s}{m}|\psi(\boldsymbol{r},t)|^2\right]\psi(\boldsymbol{r},t)$$

a_s s-wave scattering length

The GP equation is a *nonlinear Schrödinger equation*

Is GP valid for soliton phenomena?

Criterium of validity:

healing length
length scale for solitons

$$\xi=\frac{1}{\sqrt{8\pi n|a_s|}}\gg d$$

particle distance

Bibliography

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