Open Quantum Systems

Lecture I:

Dissipative Quantum Phase Transitions for Photons – Part A

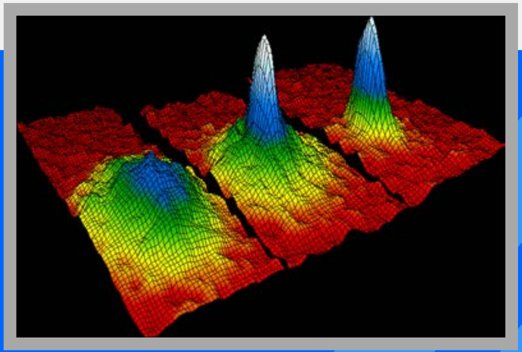
H. J. Carmichael

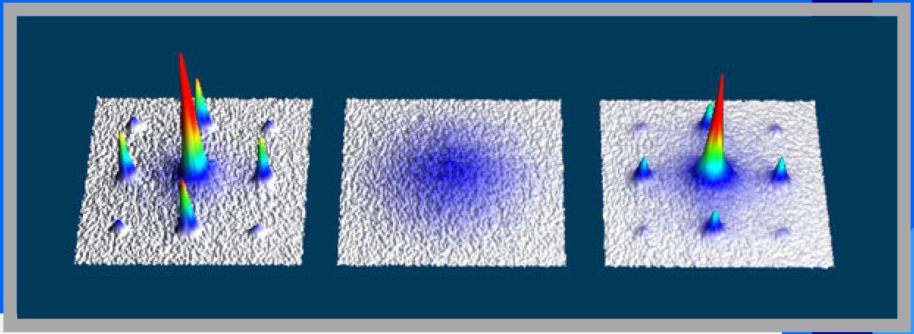
University of Auckland

Science
269, 198
(1995)

Observation of Bose-Einstein Condensation in a Dilute Atomic Vapor

M. H. Anderson,
J. R. Ensher,
M. R. Matthews,
C. E. Weiman,
E. A. Cornell





Nature
425, 39
(2002)

Quantum Phase Transition from a Superfluid to a Mott Insulator in a Gas of Ultracold Atoms

M. Greiner,
O. Mandel
T. Esslinger
T. W. Hänsch
I. Bloch

How about photons

photon number is not conserved !

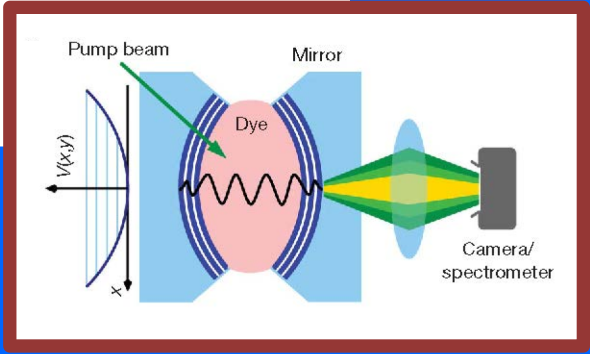
Nature 468, 545 (2010)

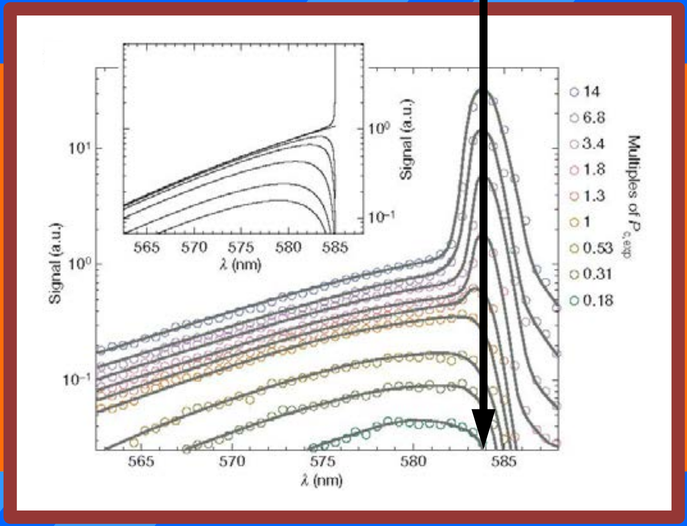
LETTER

doi:10.1038/nature09567

Bose–Einstein condensation of photons in an optical microcavity

Jan Klaers, Julian Schmitt, Frank Vewinger & Martin Weitz





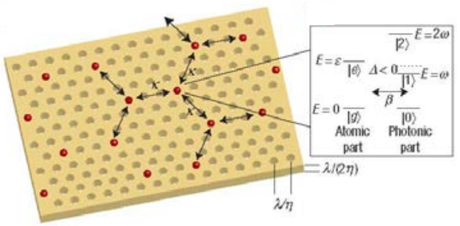
By pumping the dye with an external laser we add to a reservoir of electronic excitations that exchanges particles with the photon gas, in the sense of a grand canonical ensemble. The pumping is maintained throughout the experiment to compensate for losses …

Nature Physics 2, 856 (2006)

ARTICLES

Quantum phase transitions of light

ANDREW D. GREENTREE[1]*, CHARLES TAHAN[1,2], JARED H. COLE[1] AND LLOYD C. L. HOLLENBERG[1]
[1] Centre for Quantum Computer Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia
[2] Cavendish Laboratory, University of Cambridge, JJ Thomson Ave, Cambridge CB3 0HE, UK
* e-mail: andrew.greentree@ph.unimelb.edu.au

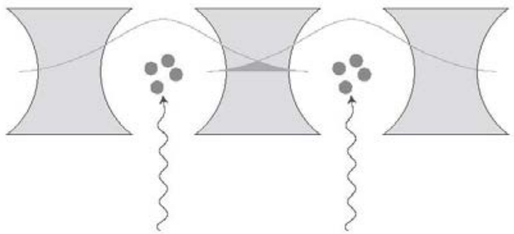


Nature Physics 2, 849 (2006)

ARTICLES

Strongly interacting polaritons in coupled arrays of cavities

MICHAEL J. HARTMANN[1,2]*, FERNANDO G. S. L. BRANDÃO[1,2] AND MARTIN B. PLENIO[1,2]*
[1] Institute for Mathematical Sciences, Imperial College London, 53 Exhibition Road, SW7 2PG, UK
[2] QOLS, The Blackett Laboratory, Imperial College London, Prince Consort Road, SW7 2BW, UK
* e-mail: m.hartmann@imperial.ac.uk; m.plenio@imperial.ac.uk



Vol 464 | 29 April 2010 | doi:10.1038/nature09009

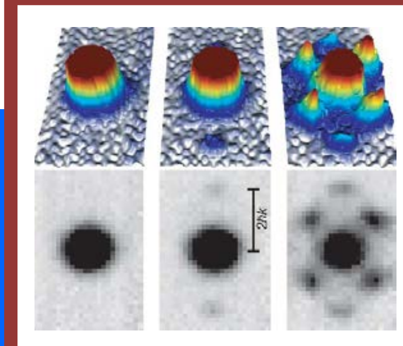
nature

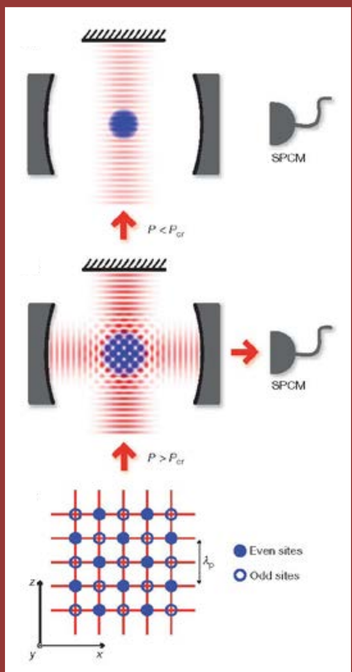
Nature 464, 1301 (2010)

ARTICLES

Dicke quantum phase transition with a superfluid gas in an optical cavity

Kristian Baumann[1], Christine Guerlin[1]†, Ferdinand Brennecke[1] & Tilman Esslinger[1]





Here we realize the Dicke quantum phase transition in an open system formed by a Bose-Einstein condensate coupled to an optical cavity, and observe the emergence of a self-organized super-solid phase. The phase transition is driven by infinitely long-range interactions between the condensate atoms, induced by two-photon processes involving the cavity mode and a pump field.

Dicke superradiant phase transition

Laser phase transition analogy – 2^{nd}-order transition

1^{st}-order transitions – optical bistabilities

Dicke superradiant phase transition

Laser phase transition analogy – 2^{nd}-order transition

1^{st}-order transitions – optical bistabilities

DICKE SUPERRADIANCE – R. H. Dicke, Phys. Rev. 93, 99 (1954)

$$H_D = \frac{\hbar\omega_{ab}}{2}\sum_{j=1}^{N}\sigma_j^z + \hbar\omega a^\dagger a + \frac{\lambda}{\sqrt{N}}\sum_{j=1}^{N}(\sigma_j^+ a + \sigma_j^- a^\dagger)$$

$$\left|\left\langle \frac{N}{2}-k,\, M-1\right|\sum_{j=1}^{N}\sigma_j^-\left|\frac{N}{2}-k,\, M\right\rangle\right|^2$$

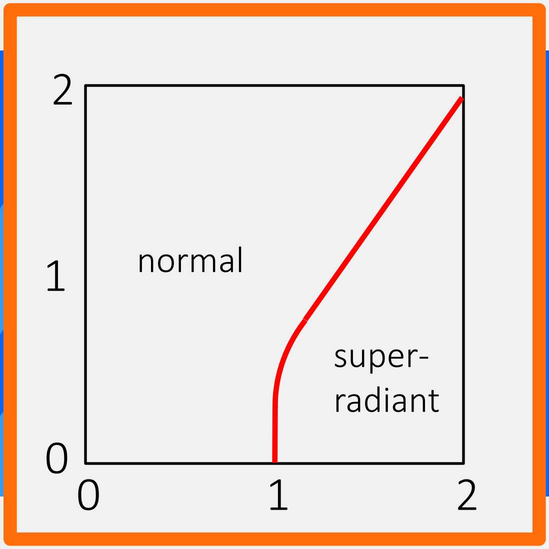
$$=\left(\frac{N}{2}-k+M\right)\left(\frac{N}{2}-k-M+1\right)$$

Annals of Physics 76, 360 (1973)

On the Superradiant Phase Transition for Molecules in a Quantized Radiation Mode: the Dicke Maser Model

K. Hepp and E. H. Lieb

$k_B T$



λ/λ_c

$4\lambda_c^2 = \omega_C \omega_A$

Physics Letters 46A, 47 (1973)

Higher Order Corrections to the Dicke Superradiant Phase Tansition

H. J. Carmichael, C. W. Gardiner, D. F. Walls

Physical Review Letters 35, 432 (1975)

Phase Transitions, Two-Level Atoms, and the A^2 Term

R. Rzążewski, K. Wódkiewicz, W. Żakowicz

We show that the presence of the recently discovered phase transition in the Dicke Hamiltonian is due entirely to the absence of the A^2 terms from the interaction Hamiltonian.

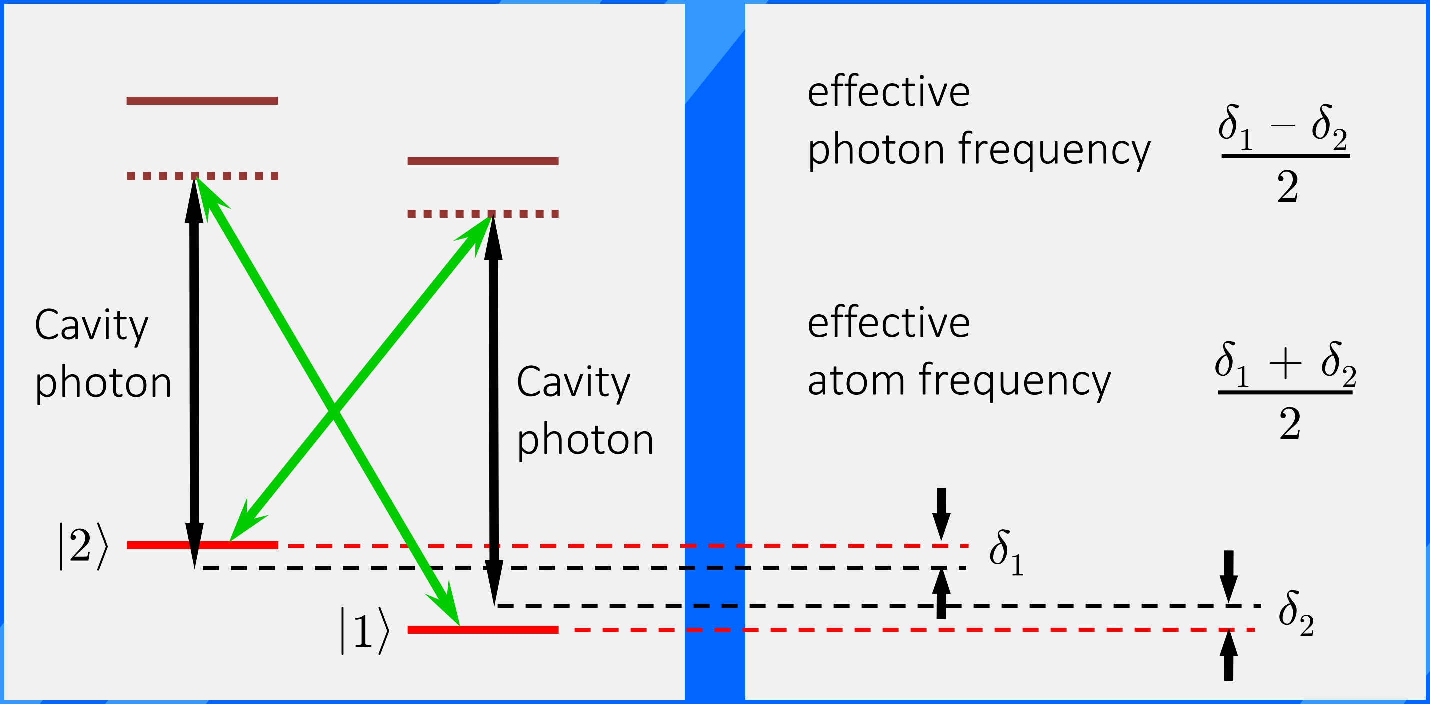
$$H_D = \frac{\hbar\omega_{ab}}{2}\sum_{j=1}^{N}\sigma_j^z + \hbar\omega a^\dagger a + \frac{\lambda}{\sqrt{N}}\sum_{j=1}^{N}(\sigma_j^+ a + \sigma_j^- a^\dagger)$$

$$H_M = \sum_{j=1}^{N}\left[\frac{1}{2m}\left(\vec{p}_j - \frac{e}{c}\vec{A}(\vec{r}_j)\right)^2 + V(\vec{r}_j)\right] + \hbar\omega a^\dagger a$$

Physical Review A 75, 013804 (2007)

Proposed Realization of the Dicke-Model Quantum Phase Transition in an optical Cavity QED System

F. Dimer, B. Estienne, A. S. Parkins, H. J. Carmichael



OPEN DICKE MODEL

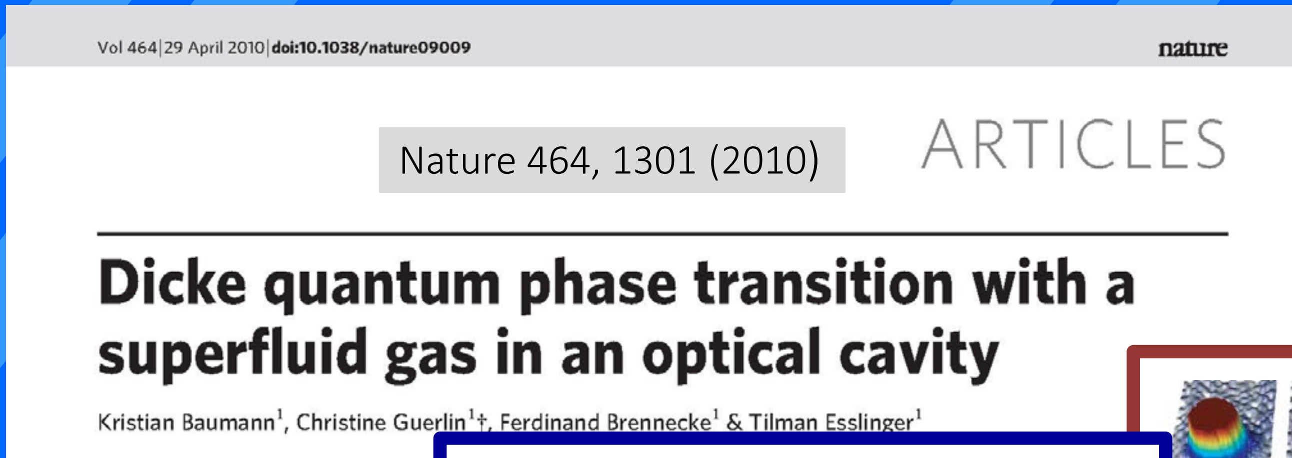
input:
coherent drive

$$H_D^{\text{eff}} =$$

$$\hbar \frac{\delta_1 + \delta_2}{4} \sum_{j=1}^{N} \sigma_j^z + \hbar \frac{\delta_1 - \delta_2}{2} a^\dagger a + \frac{\lambda}{\sqrt{N}} \sum_{j=1}^{N} (\sigma_j^+ a + \sigma_j^- a^\dagger)$$

output:
cavity loss

$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_D^{\text{eff}}, \rho] + \kappa(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a)$$

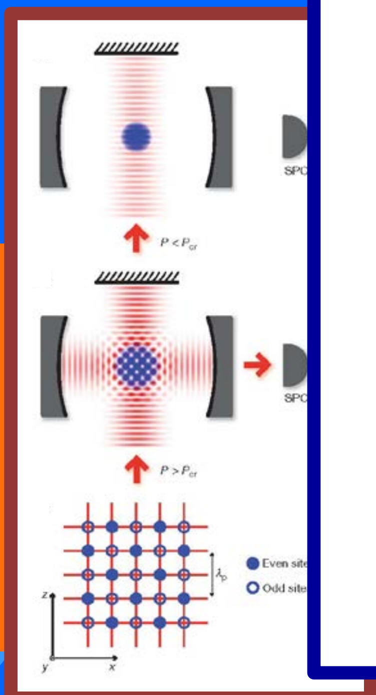

Vol 464 | 29 April 2010 | doi:10.1038/nature09009
nature

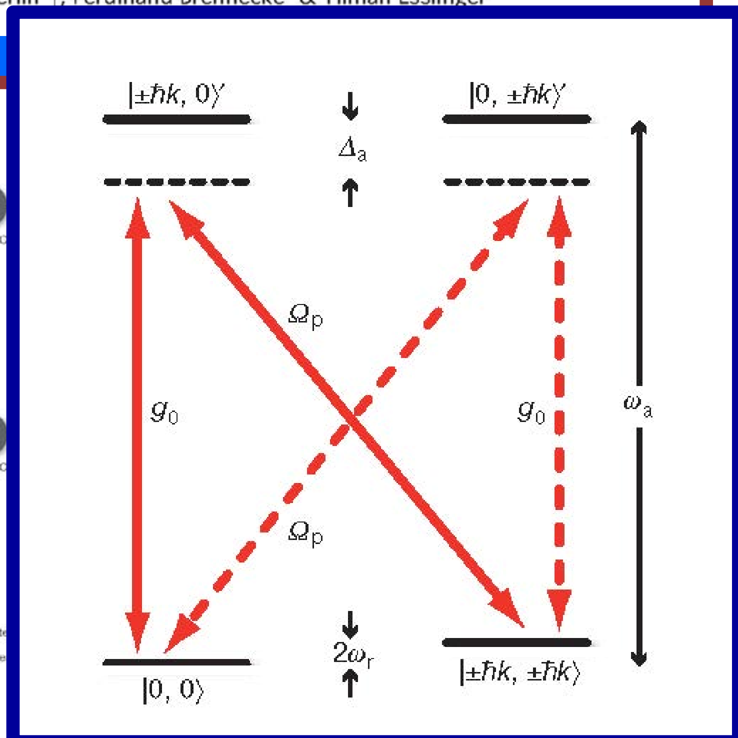
Nature 464, 1301 (2010)

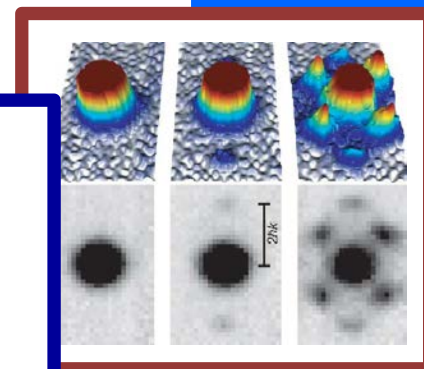
ARTICLES

Dicke quantum phase transition with a superfluid gas in an optical cavity

Kristian Baumann[1], Christine Guerlin[1]†, Ferdinand Brennecke[1] & Tilman Esslinger[1]







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Dicke superradiant phase transition

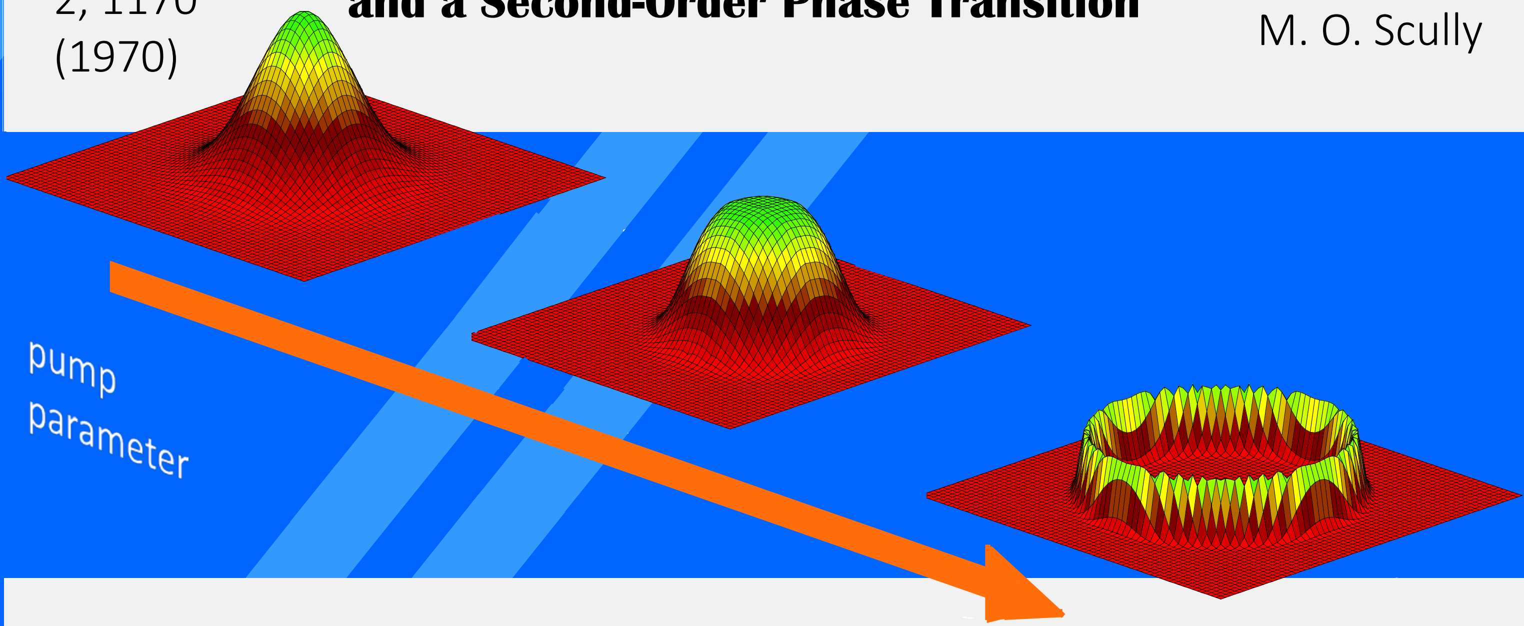
Laser phase transition analogy – 2^{nd}-order transition

1^{st}-order transitions – optical bistabilities

Physical Review A 2, 1170 (1970)

Analogy between the Laser Threshold Region and a Second-Order Phase Transition

V. DeGiorgio and M. O. Scully



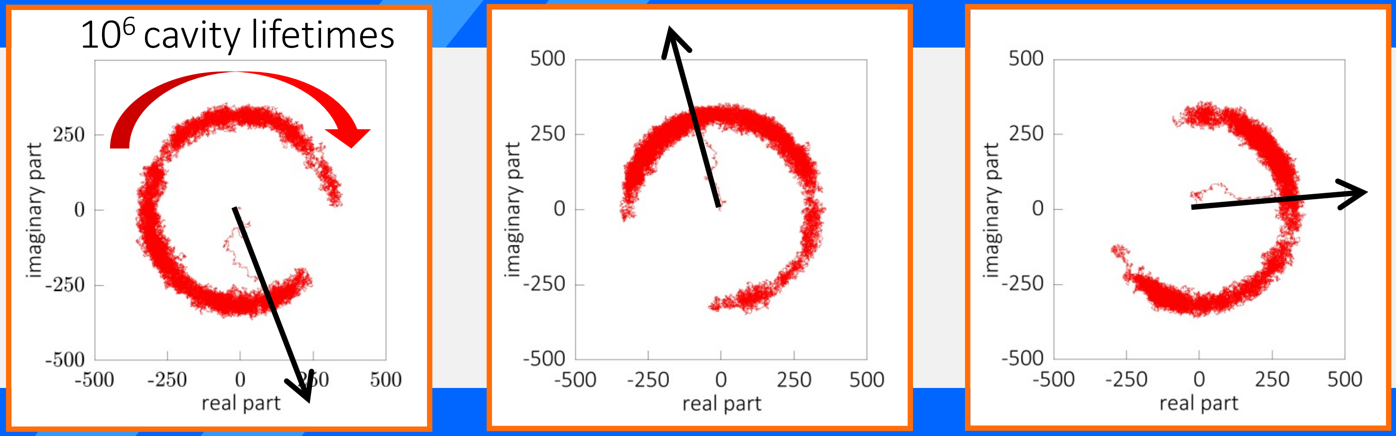
Zeitschrift für Physik 237, 31 (1970)

Laserlight—First Example of a Second-Order Phase Transition Far Away from Equilibrium

R. Graham and H. Haken

ORDER-PARAMETER, SYMMETRY BREAKING & FLUCTUATIONS

$$d\bar{\alpha} = -\bar{\alpha}(1 - p + p|\bar{\alpha}|^2)dt + \frac{1}{\sqrt{n_{\mathrm{sat}}}}(dW_1 + i dW_2)$$



$$\alpha = \sqrt{n_{\mathrm{sat}}}\;\bar{\alpha}$$

SATURATION PHOTON NUMBER & “THERMODYNAMIC” LIMIT

$$n_{\text{sat}} = \frac{\gamma_h \gamma}{4g^2}$$

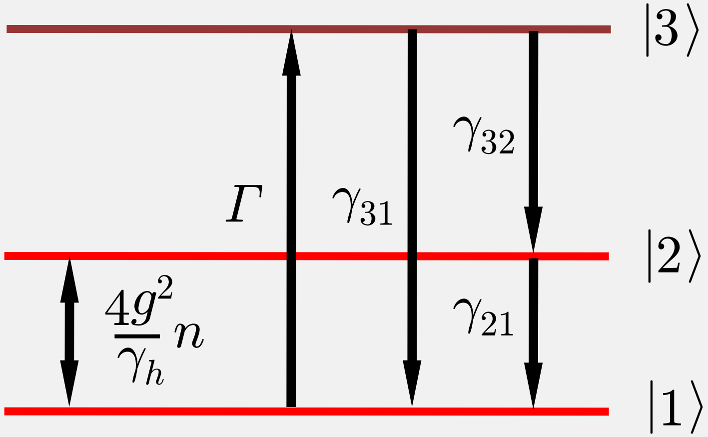
“thermodynamic” limit

a weak-coupling limit

Population inversion:

$$N_2 - N_1 = \frac{N_2^0 - N_1^0}{1 + n/n_{\text{sat}}}$$

Lasing transition:



NUMBER OF ATOMS & "THERMODYNAMIC" LIMIT

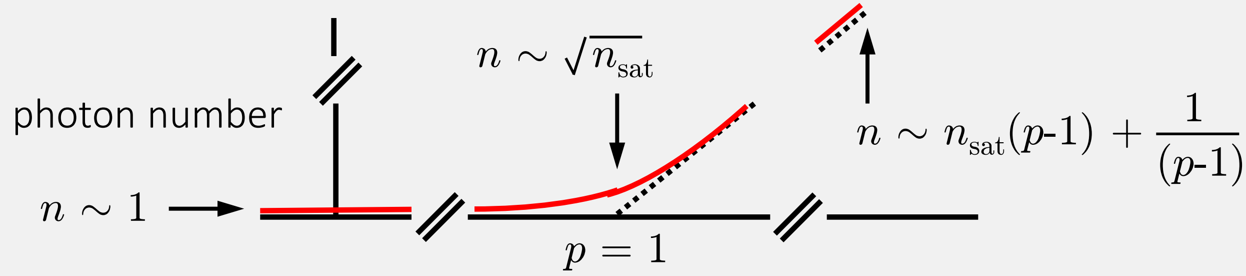
weak-coupling limit

$$g = \sqrt{\frac{\omega_C \mu^2}{2\hbar\epsilon_o V}}$$

a large volume limit

$$p = \frac{2g^2}{\gamma_h \kappa}(N_2^0 - N_1^0) \sim 1$$

a many-atom limit



Nature 482, 204 (2012)

LETTER

doi:10.1038/nature10840

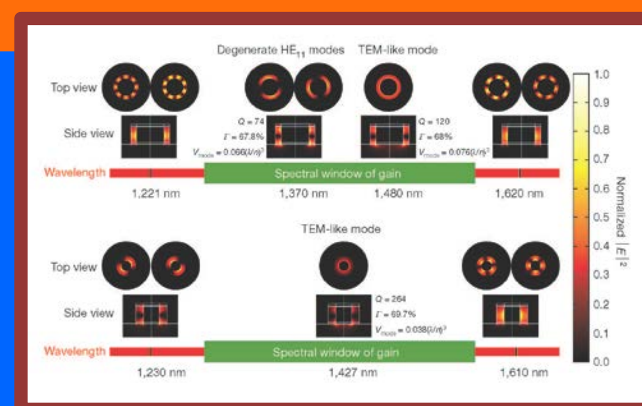
Thresholdless nanoscale coaxial lasers

M. Khajavikhan[1], A. Simic[1*], M. Katz[1*], J. H. Lee[1†], B. Slutsky[1], A. Mizrahi[1], V. Lomakin[1] & Y. Fainman[1]

A $\beta = 0.95$ laser

$$\beta = \frac{4g^2/\gamma_h}{\gamma} = \frac{4g^2/\gamma_h}{\gamma_{\text{loss}} + 4g^2/\gamma_h}$$

$$n_{\text{sat}} = \beta^{-1} \geq 1$$

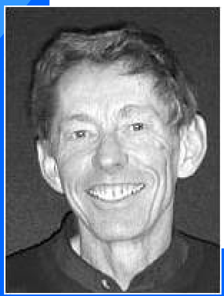


Dicke superradiant phase transition

Laser phase transition analogy – 2nd-order transition

1st-order transitions – optical bistabilities

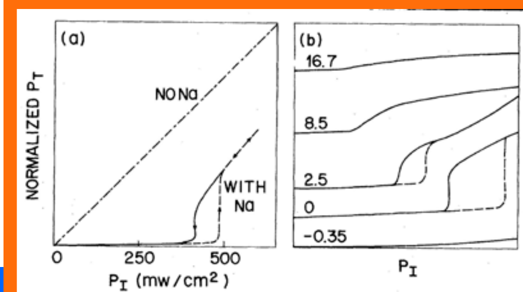
VOLUME 36, NUMBER 19 PHYSICAL REVIEW LETTERS 10 MAY 1976



Differential Gain and Bistability Using a Sodium-Filled Fabry-Perot Interferometer

H. M. Gibbs,* S. L. McCall, and T. N. C. Venkatesan†
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 9 February 1976)

Differential gain and large hysteresis have been seen in the transmission of a Fabry-Perot interferometer containing Na vapor irradiated by light from a cw dye laser. Nonlinear dispersion, neglected in earlier work, dominates over nonlinear absorption in Na. The apparatus uses only optical inputs and outputs. Similar apparatus may be useful as an optical amplifier, memory element, clipper, and limiter.



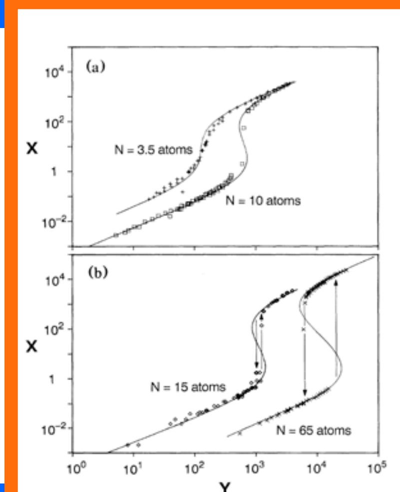
VOLUME 67, NUMBER 13 PHYSICAL REVIEW LETTERS 23 SEPTEMBER 1991

Optical Bistability and Photon Statistics in Cavity Quantum Electrodynamics

G. Rempe, R. J. Thompson, R. J. Brecha,[a] W. D. Lee,[b] and H. J. Kimble
Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125
(Received 28 June 1991)

The quantum statistical behavior of a small collection of N two-state atoms strongly coupled to the field of a high-finesse optical cavity is investigated. Input-output characteristics are recorded over the range $3 \lesssim N \lesssim 65$, with bistability observed for $N \gtrsim 15$ intracavity atoms and for a saturation photon number $n_0 \simeq 0.8$. For weak excitation the transmitted field exhibits photon antibunching as a nonclassical manifestation of state reduction and quantum interference with the magnitude of the nonclassical effects largely independent of N.

PACS numbers: 42.50.Kb, 32.80.−t, 42.50.Dv, 42.65.Pc



MAXWELL-BLOCH EQUATIONS

$$\frac{d\alpha}{dt} = -(\kappa - i\Delta\omega_C)\alpha + Ng\beta + \mathcal{E}$$

cavity field

$$\frac{d\beta}{dt} = -(\gamma_h/2 - i\Delta\omega_A)\beta + g\alpha\zeta$$

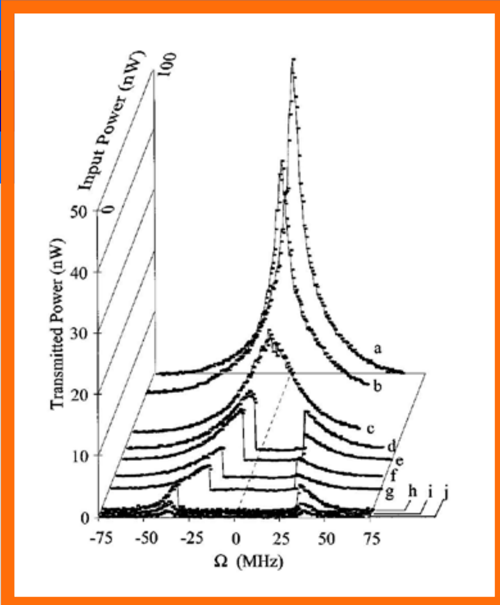
atomic polarization

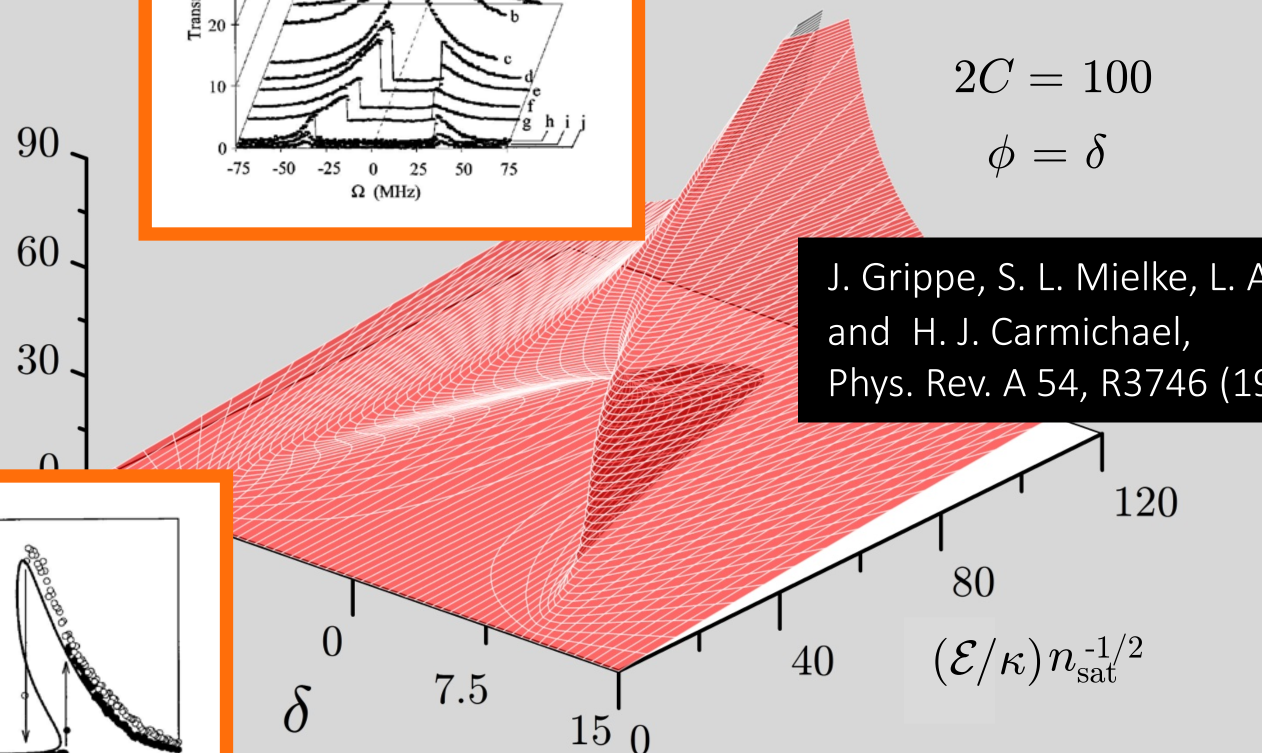
$$\frac{d\zeta}{dt} = -\gamma(\zeta + 1) - 2g(\alpha^*\beta + \alpha\beta^*)$$

atomic inversion

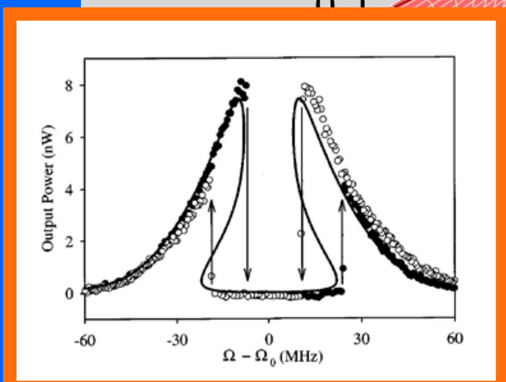
$$\frac{(\mathcal{E}/\kappa)^2}{n_{\mathrm{sat}}} = \frac{n}{n_{\mathrm{sat}}}\left[\left(1 + \frac{2C}{1+\delta^2+n/n_{\mathrm{sat}}}\right)^2 + \left(\phi - \frac{2C\delta}{1+\delta^2+n/n_{\mathrm{sat}}}\right)^2\right]$$

$$C = Ng^2/\gamma_h\kappa \qquad \delta = 2\Delta\omega_A/\gamma_h \qquad \phi = \Delta\omega_C/\kappa$$





J. Grippe, S. L. Mielke, L. A. Orozco, and H. J. Carmichael, Phys. Rev. A 54, R3746 (1996)



PRL **111**, 113901 (2013) PHYSICAL REVIEW LETTERS week ending 13 SEPTEMBER 2013

Nonequilibrium Phase Transition in a Dilute Rydberg Ensemble

C. Carr, R. Ritter, C. G. Wade, C. S. Adams, and K. J. Weatherill
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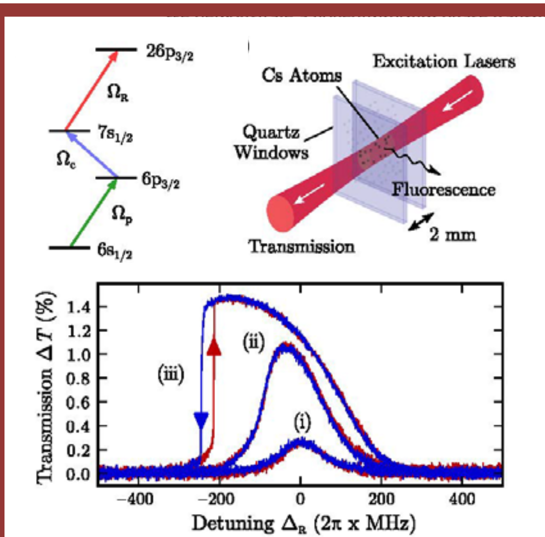
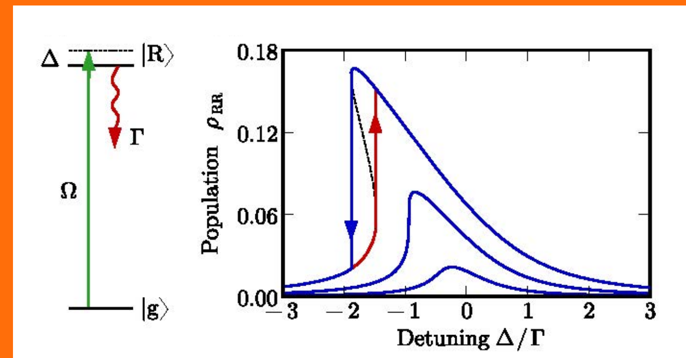
(Received 28 March 2013; published 10 September 2013)

n in a dilute thermal atomic gas. The phase transition,
cy, is induced by resonant dipole-dipole interactions
as dilute as the atoms are separated by distances much
ns used to excite them. In the frequency domain, we
h results in intrinsic optical bistability above a critical
erve critical slowing down where the recovery time to
t $\alpha = -0.53 \pm 0.10$. The atomic emission spectrum
s evidence for a super

PACS numbers: 42.6





Physical Review A 19, 2392 (1979)

First and Second-Order Phase Transitions in the Dicke Model: Relation to Optical Bistability

C. M. Bowden and C. C. Sung

Open Quantum Systems

Lecture II:

Dissipative Quantum Phase Transitions for Photons – Part B

H. J. Carmichael

University of Auckland

Nature 482, 204 (2012)

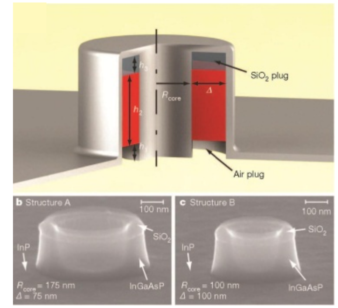
LETTER

doi:10.1038/nature10840

Thresholdless nanoscale coaxial lasers

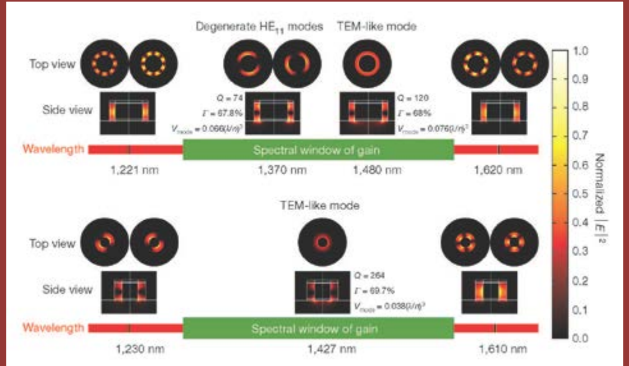
M. Khajavikhan[1], A. Simic[1]*, M. Katz[1]*, J. H. Lee[1]†, B. Slutsky[1], A. Mizrahi[1], V. Lomakin[1] & Y. Fainman[1]

A $\beta = 0.95$ laser



$$\beta = \frac{4g^2/\gamma_h}{\gamma} = \frac{4g^2/\gamma_h}{\gamma_{\text{loss}} + 4g^2/\gamma_h}$$

$$n_{\text{sat}} = \beta^{-1} \geq 1$$

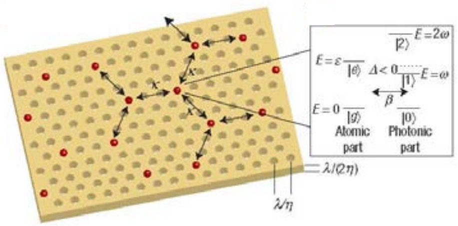


Nature Physics 2, 856 (2006)

ARTICLES

Quantum phase transitions of light

ANDREW D. GREENTREE[1]*, CHARLES TAHAN[1,2], JARED H. COLE[1] AND LLOYD C. L. HOLLENBERG[1]
[1]Centre for Quantum Computer Technology, School of Physics, The University of Melbourne, Victoria 3010, Australia
[2]Cavendish Laboratory, University of Cambridge, JJ Thomson Ave, Cambridge CB3 0HE, UK
*e-mail: andrew.greentree@ph.unimelb.edu.au

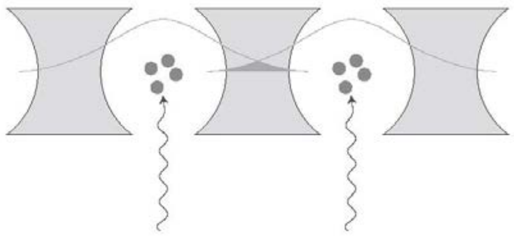


Nature Physics 2, 849 (2006)

ARTICLES

Strongly interacting polaritons in coupled arrays of cavities

MICHAEL J. HARTMANN[1,2]*, FERNANDO G. S. L. BRANDÃO[1,2] AND MARTIN B. PLENIO[1,2]*
[1]Institute for Mathematical Sciences, Imperial College London, 53 Exhibition Road, SW7 2PG, UK
[2]QOLS, The Blackett Laboratory, Imperial College London, Prince Consort Road, SW7 2BW, UK
*e-mail: m.hartmann@imperial.ac.uk; m.plenio@imperial.ac.uk



Strong coupling and photon blockade

1st- and 2nd-order transitions in the open JC model

Openness, observation & fluctuations

Strong coupling and photon blockade

1st- and 2nd-order transitions in the open JC model

Openness, entanglement, observation & fluctuations

VOLUME 79, NUMBER 8 PHYSICAL REVIEW LETTERS 25 AUGUST 1997

Strongly Interacting Photons in a Nonlinear Cavity

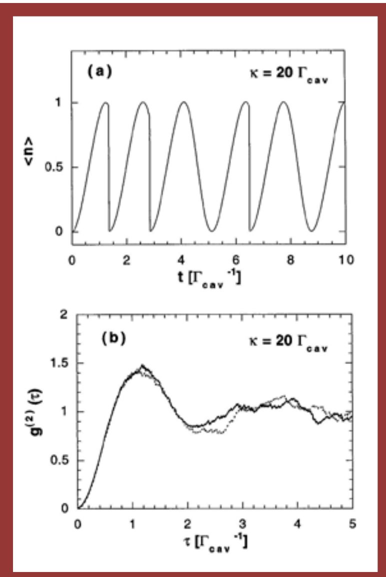
A. Imamoḡlu,[1] H. Schmidt,[1] G. Woods,[1] and M. Deutsch[2]
[1]*Department of Electrical and Computer Engineering, University of California, Santa Barbara, California 93106*
[2]*Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544*

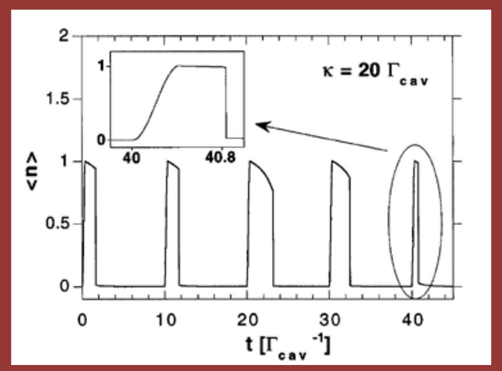
(Received 12 November 1996)

We consider the dynamics of single photons in a nonlinear optical cavity. When the Kerr nonlinearities of *atomic dark resonances* are utilized, the cavity mode is well described by a spin-1/2 Hamiltonian. We show that it is possible to achieve coherent control of the cavity-mode wave function using π pulses for single photons that switch the state of the cavity with very high accuracy. The underlying physics is best understood as the nonlinearity induced anticorrelation between single-photon injection/emission events, which we refer to as *photon blockade*. We also propose a method which uses these strong dispersive interactions to realize a single-photon turnstile device. [S0031-9007(97)03903-3]

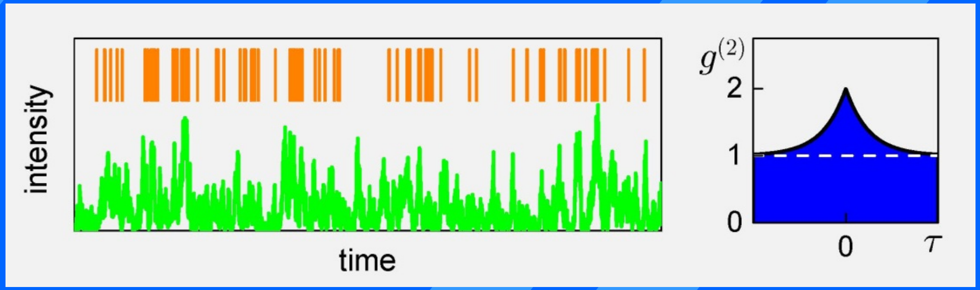
PACS numbers: 42.50.Dv, 03.65.Bz, 42

To explain the strong antibunching of transmitted photons, we introduce the concept of *photon blockade* in close analogy with the phenomenon of Coulomb blockade for quantum-well electrons.

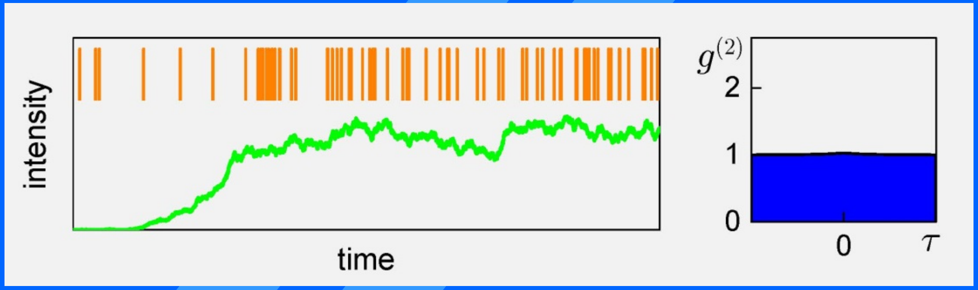




PHOTON ANTIBUNCHING



bunched light
e.g. laser below threshold



coherent light
e.g. laser above threshold

antibunched light
e.g. resonance fluorescence

RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 46, NUMBER 11 1 DECEMBER 1992

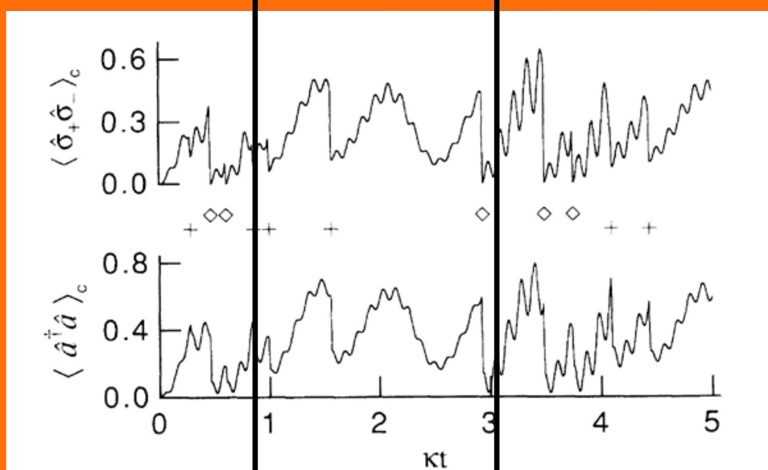
Quantum trajectory simulations of the two-state behavior of an optical cavity containing one atom

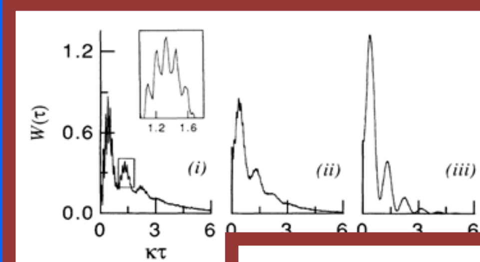
L. Tian and H. J. Carmichael
Department of Physics, Chemical Physics Institute, and Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403

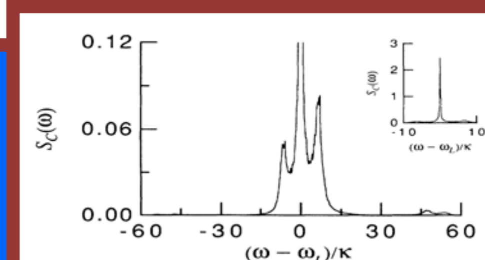
(Received 26 August 1992)

Under conditions of strong dipole coupling an optical cavity containing one atom acts as a two-state system when excited near one of the "vacuum" Rabi resonances.

L. Tian and H. J. Carmichael, Phys. Rev. A 46, R6801 (1992)



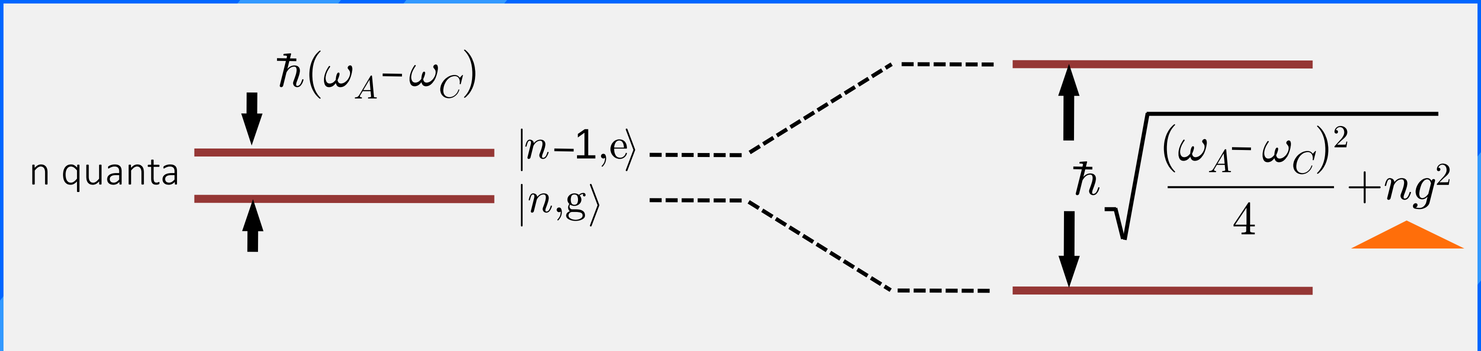




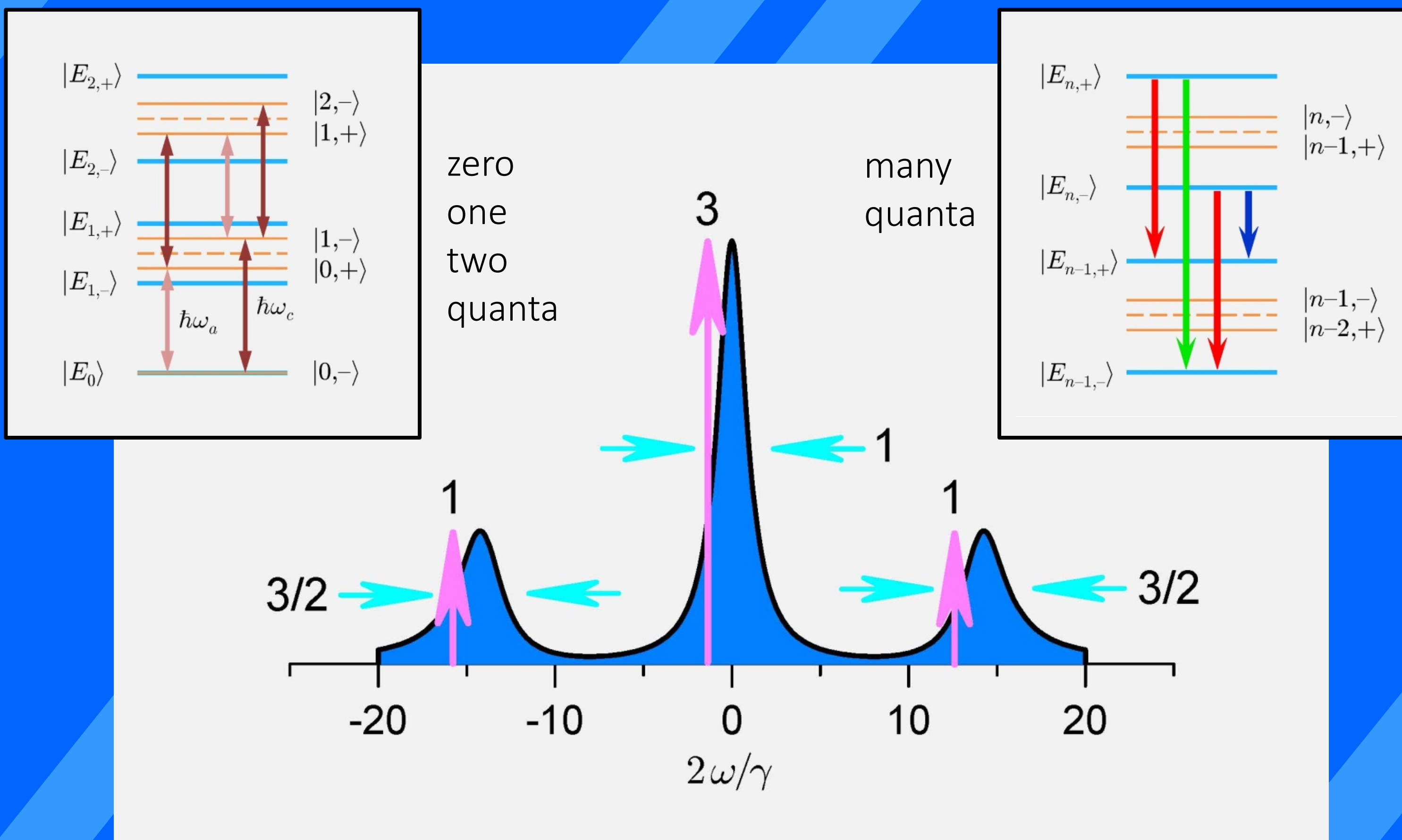
JAYNES-CUMMINGS MODEL

$$H_{JC} = \hbar\omega_C a^\dagger a + \frac{\hbar\omega_A}{2}(|e\rangle\langle e| - |g\rangle\langle g|)$$ free cavity + atom

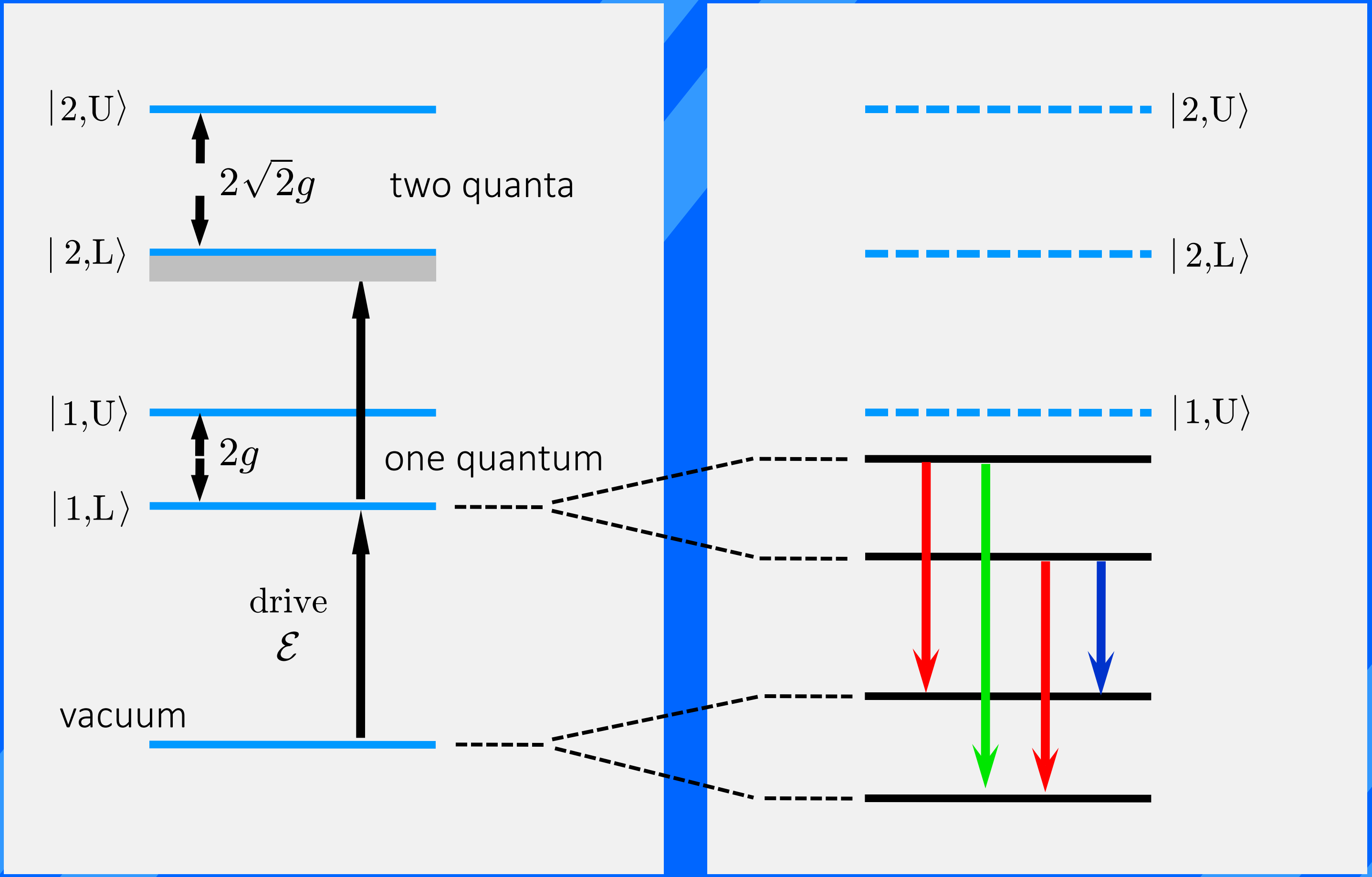
$$+ \hbar g(a^\dagger|g\rangle\langle e| + a|e\rangle\langle g|)$$ dipole coupling



DRESSED STATES & THE MOLLOW TRIPLET



STRONG COUPLING – DRESSING THE DRESSED STATES



Vol 436|7 July 2005|doi:10.1038/nature03804 nature

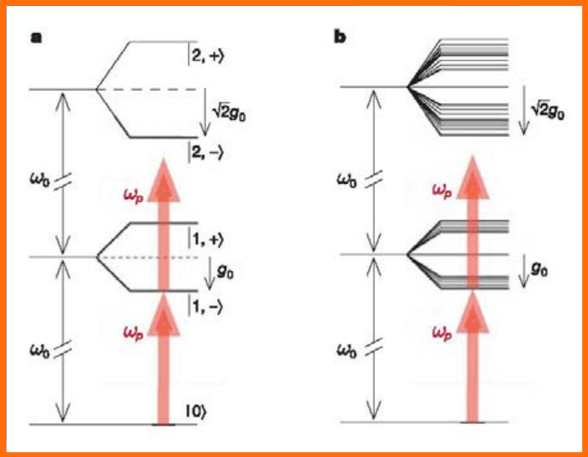
Nature 436, 87 (2005)

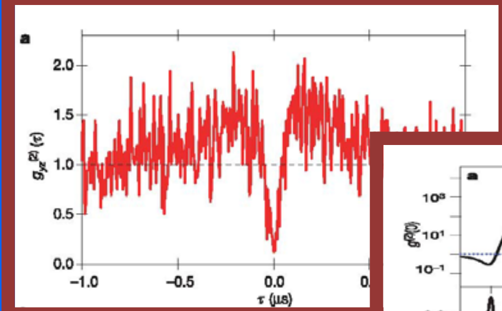
LETTERS

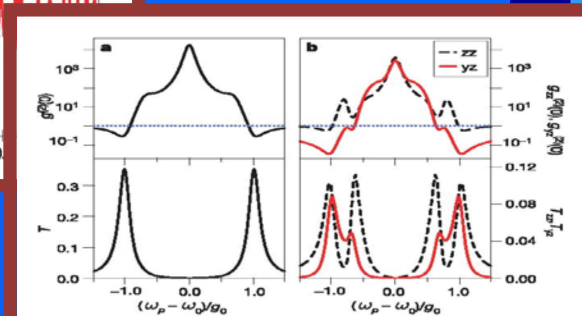
Photon blockade in an optical cavity with one trapped atom

K. M. Birnbaum[1], A. Boca[1], R. Miller[1], A. D. Boozer[1], T. E. Northup[1] & H. J. Kimble[1]

(coupling, cavity width, atom width) (34 , 4.1, 2.6) MHz



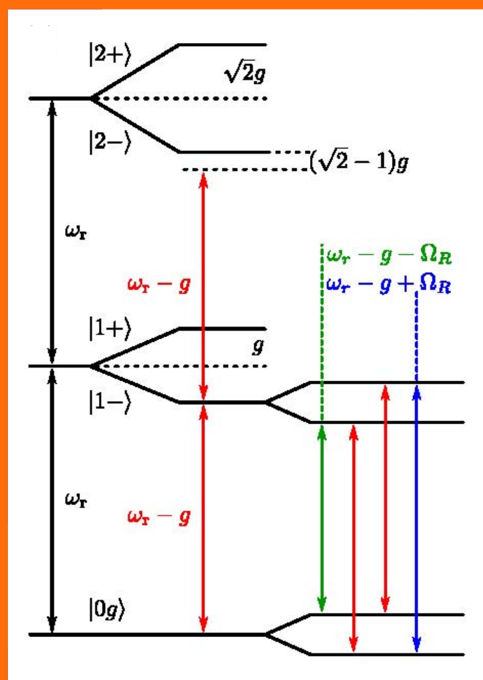


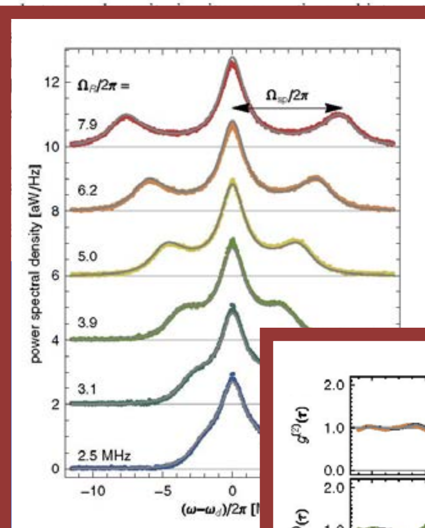


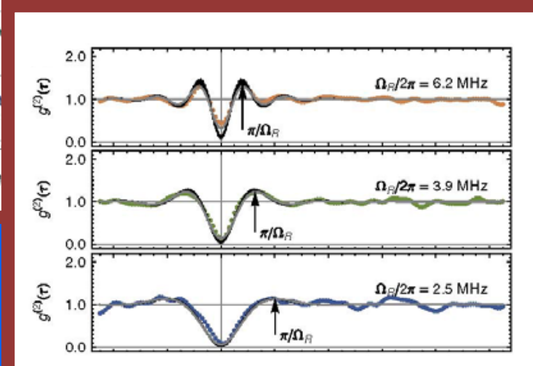
PRL 106, 243601 (2011) PHYSICAL REVIEW LETTERS week ending 17 JUNE 2011

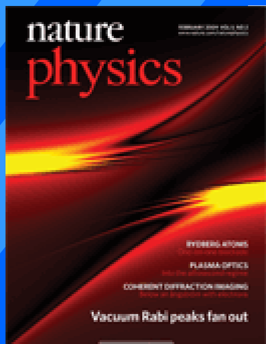
Observation of Resonant Photon Blockade at Microwave Frequencies Using Correlation Function Measurements

C. Lang,[1] D. Bozyigit,[1] C. Eichler,[1] L. Steffen,[1] J. M. Fink,[1] A. A. Abdumalikov, Jr.,[1] M. Baur,[1] S. Filipp,[1] M. P. da Silva,[2] A. Blais,[2] and A. Wallraff[1]
[1]Department of Physics, ETH Zürich, CH-8093, Zürich, Switzerland
[2]Département de Physique, Université de Sherbrooke, Sherbrooke, Québec, J1K 2R1 Canada
(Received 17 March 2011; published 15 June 2011)





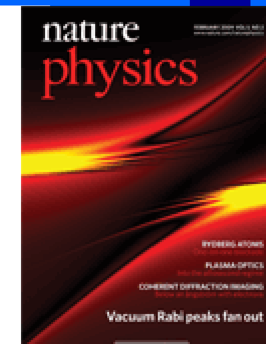




Nature Physics 5, 105 (2009)

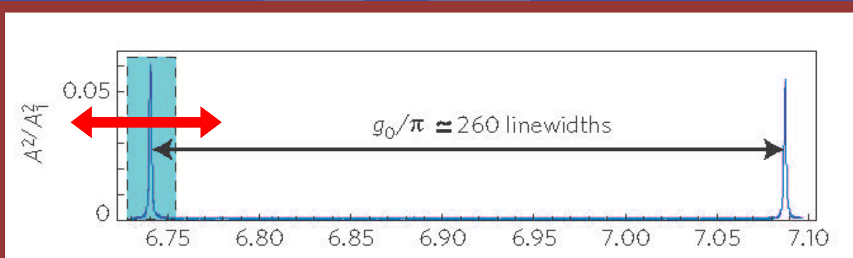
Nonlinear response of the vacuum Rabi resonance

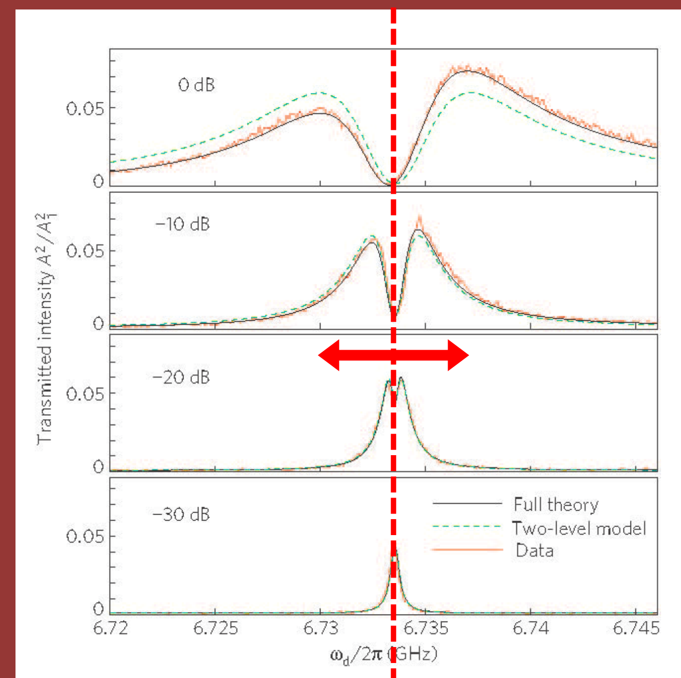
Lev S. Bishop[1], J. M. Chow[1], Jens Koch[1], A. A. Houck[1], M. H. Devoret[1], E. Thuneberg[2], S. M. Girvin[1] and R. J. Schoelkopf[1]*



field amplitude

$$\frac{1 + i\delta}{1 + n/n_{\mathrm{sat}} + \delta^2}$$





Strong coupling and photon blockade

1st- and 2nd-order transitions in the open JC model

Openness, entanglement, observation & fluctuations

Physical Review A 2, 1170 (1970)

Analogy between the Laser Threshold Region and a Second-Order Phase Transition

V. DeGiorgio and M. O. Scully

saturable nonlinearity

$$N_2 - N_1 = \frac{N_2^0 - N_1^0}{1 + n/n_{\text{sat}}}$$

"thermodynamic" limit:

$$n_{\text{sat}} = \frac{\gamma_h \gamma}{4g^2} \to \infty$$

weak-coupling limit

Zeitschrift für Physik 237, 31 (1970)

Laserlight—First Example of a Second-Order Phase Transition Far Away from Equilibrium

R. Graham and H. Haken

Quantum Optics 3, 13 (1991)

Spontaneous Dressed-State Polarization of a Coupled Atom and Cavity Mode

P. Alsing and H. J. Carmichael

$$n/n_s = \frac{(2\,\mathcal{E}/g)^2}{1 + n_s/n}$$

"thermodynamic" limit:

$$n_s = \frac{g^2}{4\kappa^2} \to \infty$$

strong-coupling limit

CQ10 Rochester (2013)

Breakdown of Photon Blockade: A Dissipative Quantum Phase Transition in Zero Dimensions

H. J. Carmichael

DRIVEN JAYNES-CUMMINGS MODEL

$$H_{JC}^{\text{driven}}(t) = \hbar\omega_C a^\dagger a + \frac{\hbar\omega_A}{2}(|\mathrm{e}\rangle\langle\mathrm{e}| - |\mathrm{g}\rangle\langle\mathrm{g}|)$$ free cavity + atom

$$+\ \hbar g(a^\dagger|\mathrm{g}\rangle\langle\mathrm{e}| + a|\mathrm{e}\rangle\langle\mathrm{g}|)$$ dipole coupling

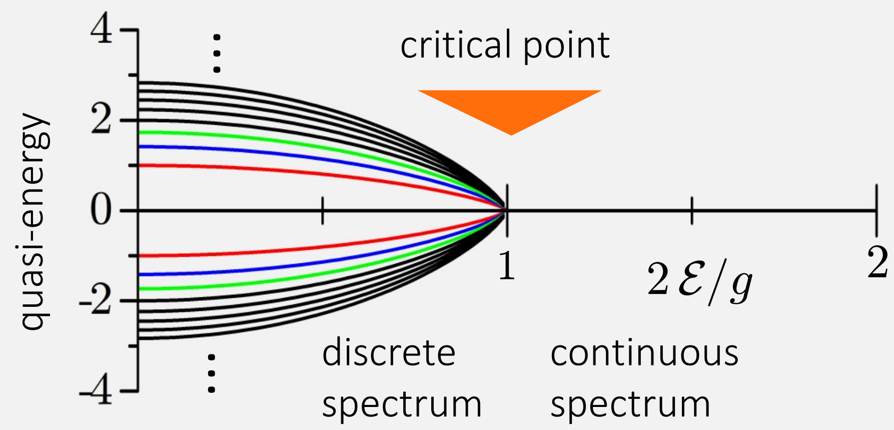
$$+\ \hbar\mathcal{E}\left(ae^{i\omega t} + a^\dagger e^{-i\omega t}\right)$$ drive

RESONANCE — $\omega = \omega_C = \omega_A$ — PLUS INTERACTION PICTURE:

$$H_{JC}^{\text{driven}} = \hbar g(a^\dagger|\mathrm{g}\rangle\langle\mathrm{e}| + a|\mathrm{e}\rangle\langle\mathrm{g}|) + \hbar\,\mathcal{E}(a + a^\dagger)$$

QUASI-ENERGIES

$$E^n_{U,L} = \pm\sqrt{n}\hbar g\left[1 - \left(\frac{2\mathcal{E}}{g}\right)^2\right]^{3/4}$$



MASTER EQUATION – OPEN JAYNES-CUMMINGS MODEL

input:
coherent drive

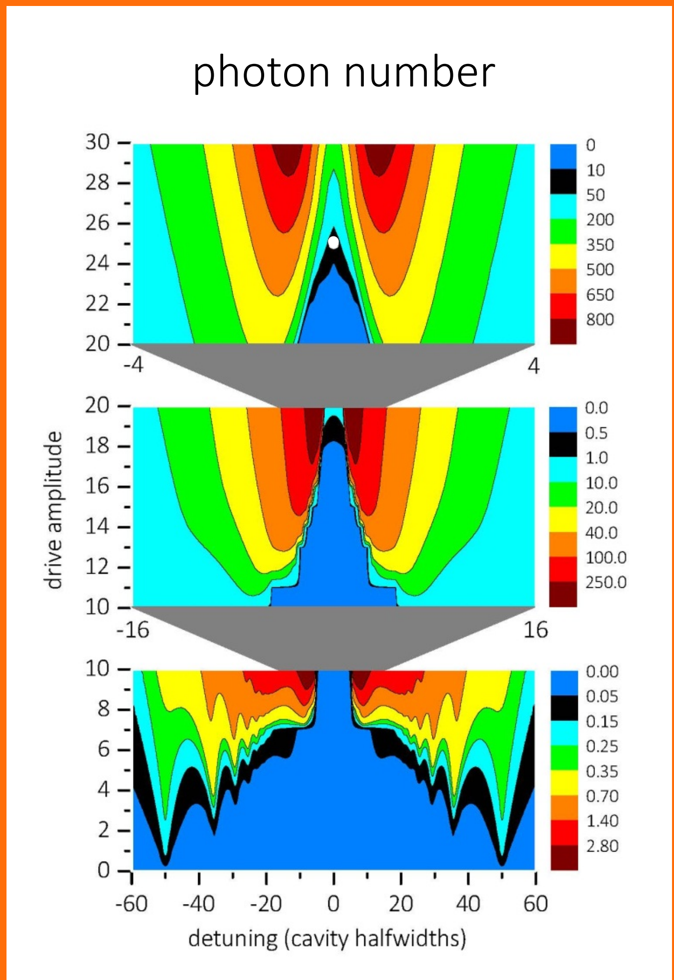
output:
cavity loss

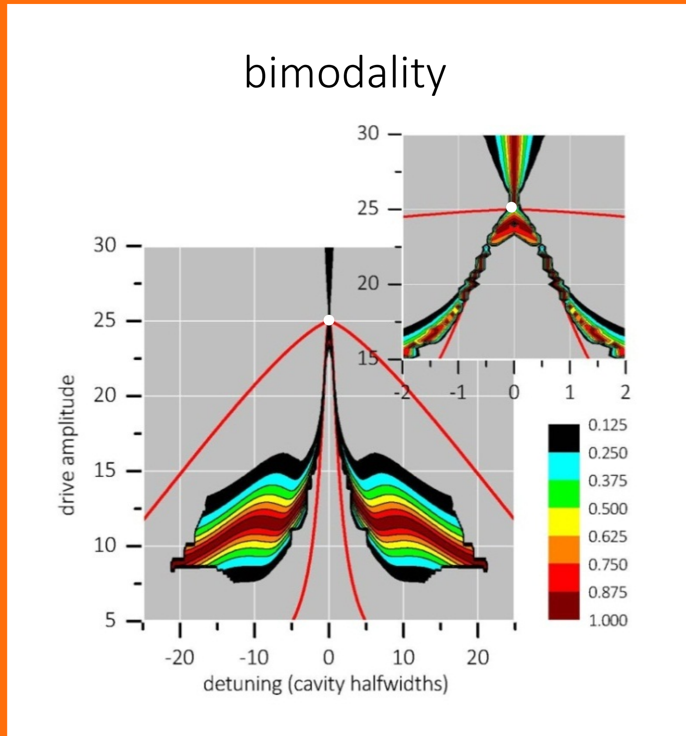
$$\frac{d\rho}{dt} = \frac{1}{i\hbar}[H_{JC}^{\text{driven}}(t), \rho] + \kappa(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

output:
spontaneous emission

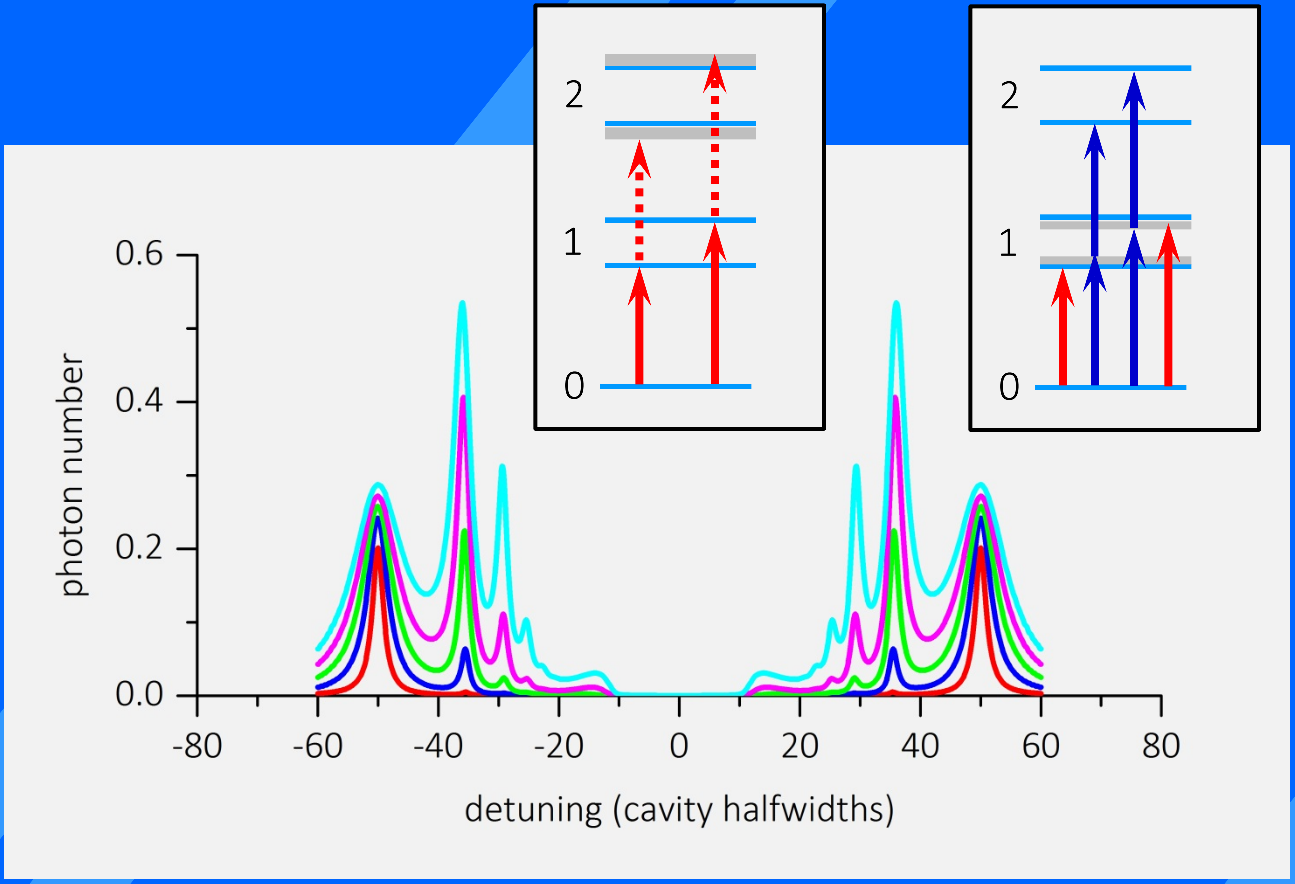
$$+\frac{\gamma}{2}(2|\mathrm{g}\rangle\langle\mathrm{e}|\rho|\mathrm{e}\rangle\langle\mathrm{g}| - |\mathrm{e}\rangle\langle\mathrm{e}|\rho - \rho|\mathrm{e}\rangle\langle\mathrm{e}|)$$

dipole coupling $= 50 \times$ cavity linewidth

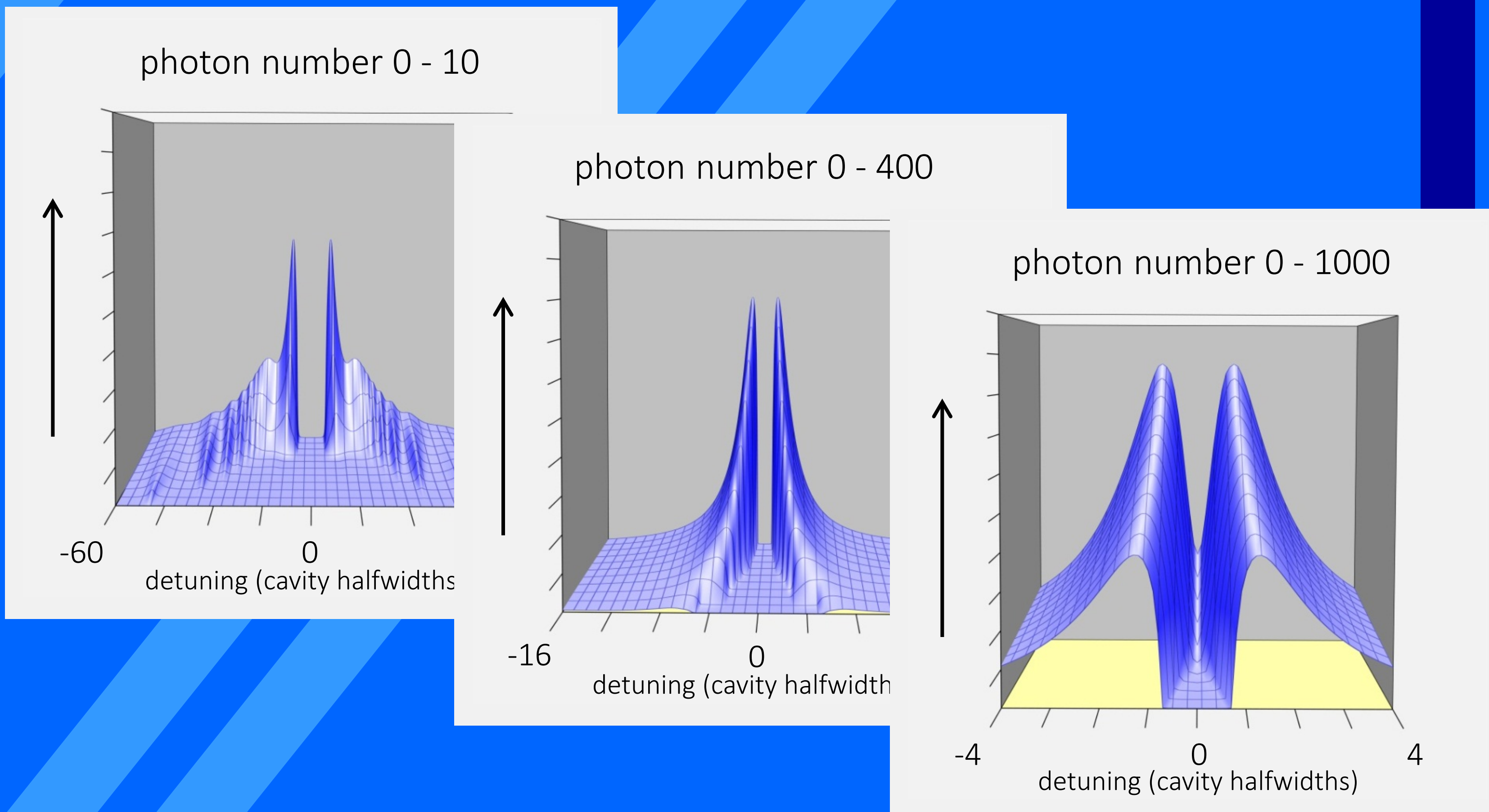




MULTIPHOTON RESONANCE & BLOCKADE



BREAKDOWN OF BLOCKADE



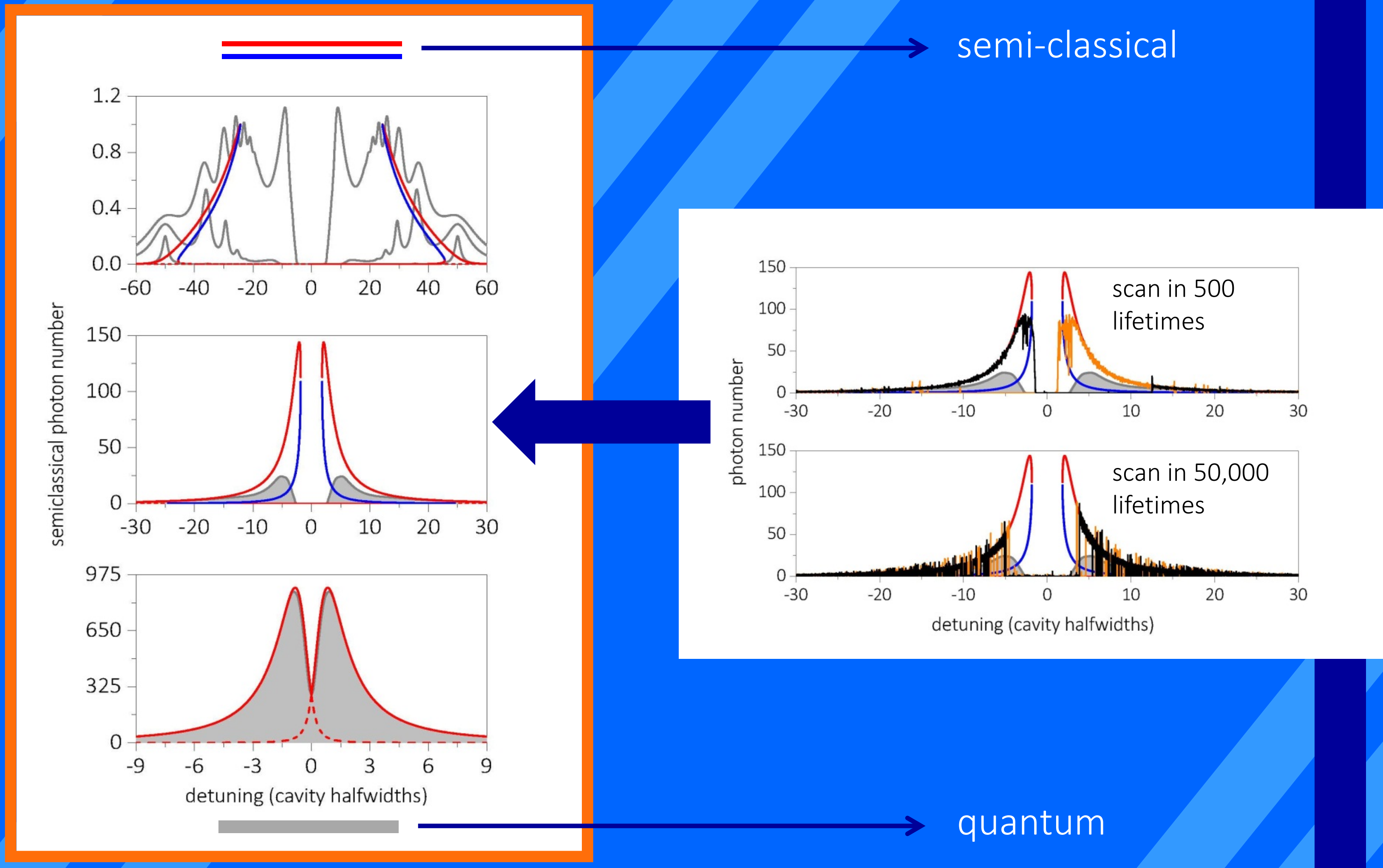
MEAN-FIELD THEORY – MAXWELL-BLOCH EQUATIONS

$$\frac{d\alpha}{dt} = -(\kappa - i\Delta\omega)\alpha + g\beta + \mathcal{E}$$ cavity field

$$\frac{d\beta}{dt} = i\Delta\omega\beta + g\alpha\zeta$$ x-spin – i y-spin

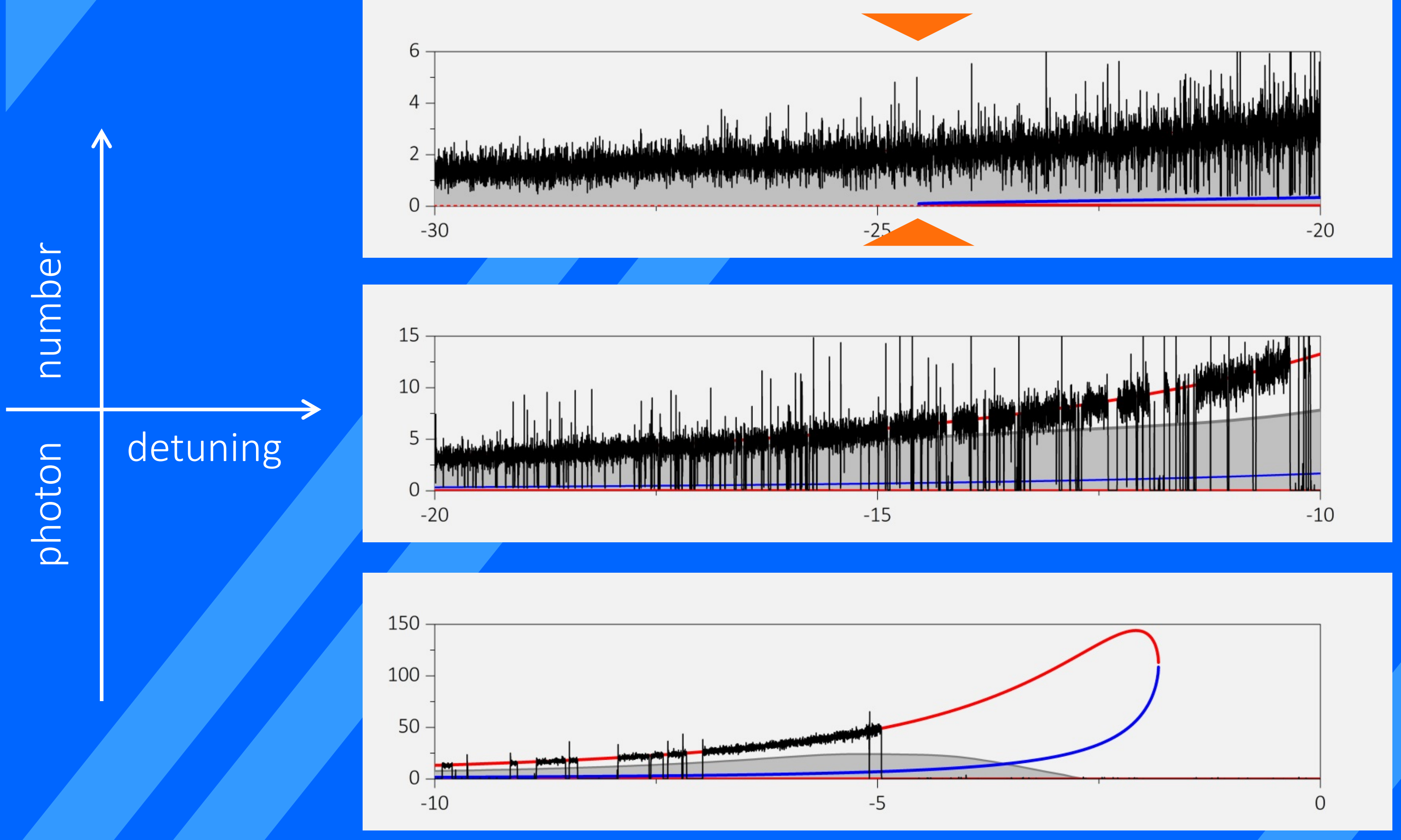
$$\frac{d\zeta}{dt} = -2g(\alpha^*\beta + \alpha\beta^*)$$ z-spin

$$\alpha = \frac{\mathcal{E}}{\kappa - i\left[\Delta\omega \mp \mathrm{sgn}(\Delta\omega)\frac{g^2}{\sqrt{\Delta\omega^2 + 4g^2|\alpha|^2}}\right]}$$



semi-classical
semiclassical photon number
1.2
0.8
0.4
0.0
-60
-40
-20
0
20
40
60
150
100
50
0
-30
-20
-10
0
10
20
30
975
650
325
0
-9
-6
-3
0
3
6
9
detuning (cavity halfwidths)
photon number
scan in 500
lifetimes
scan in 50,000
lifetimes
detuning (cavity halfwidths)
quantum

QUANTUM TRAJECTORY SIMULATION



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$$\frac{d\alpha}{dt} = \underline{-(\kappa - i\Delta\omega)\alpha + g\beta + \mathcal{E}}$$ cavity field

$$\frac{d\beta}{dt} = \underline{i\Delta\omega\beta + g\alpha\zeta} - \frac{\gamma}{2}\beta$$ x-spin – i y-spin

$$\frac{d\zeta}{dt} = \underline{-2g(\alpha^*\beta + \alpha\beta^*)} - \gamma(\zeta+1)$$ z-spin

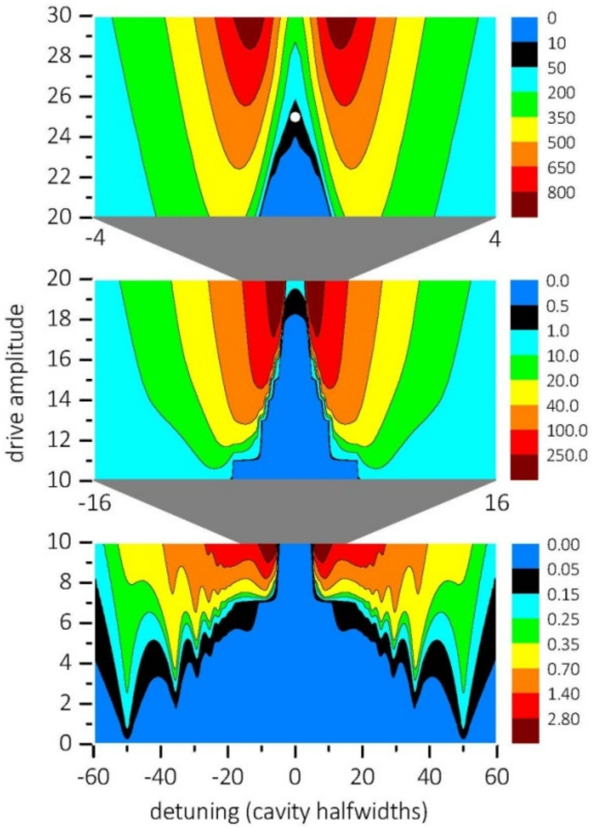
$$\frac{(\mathcal{E}/\kappa)^2}{n_{\text{sat}}} = \frac{n}{n_{\text{sat}}}\left[\left(1 + \frac{2C}{1+\delta^2+n/n_{\text{sat}}}\right)^2 + \left(\phi - \frac{2C\delta}{1+\delta^2+n/n_{\text{sat}}}\right)^2\right]$$

$C = g^2/\gamma\kappa$ $\quad$ $\delta = 2\Delta\omega/\gamma$ $\quad$ $\phi = \Delta\omega/\kappa$

dipole coupling $= 50 \times$ cavity linewidth

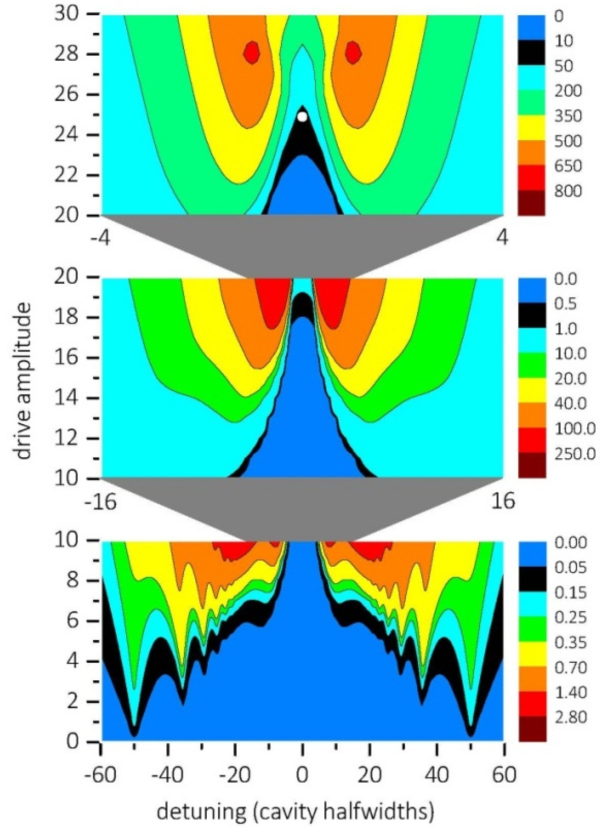
$\gamma/\kappa = 0$

$n_{\mathrm{sat}} = 0$



$\gamma/\kappa = 0.1$

$n_{\mathrm{sat}} = 5\times10^{-7}$



$\gamma/\kappa = 1.0$

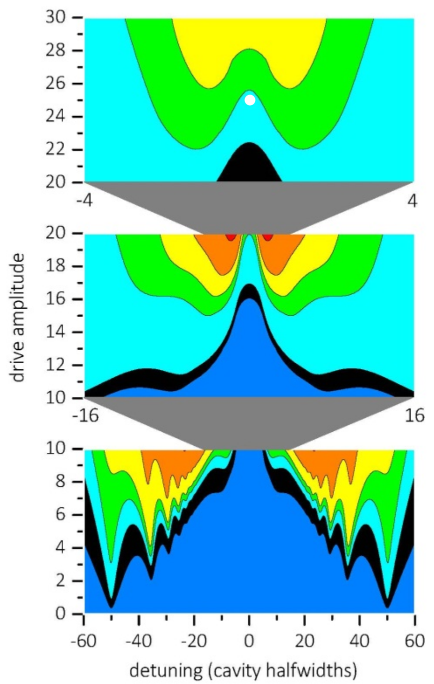
$n_{\text{sat}} = 5\times10^{-5}$

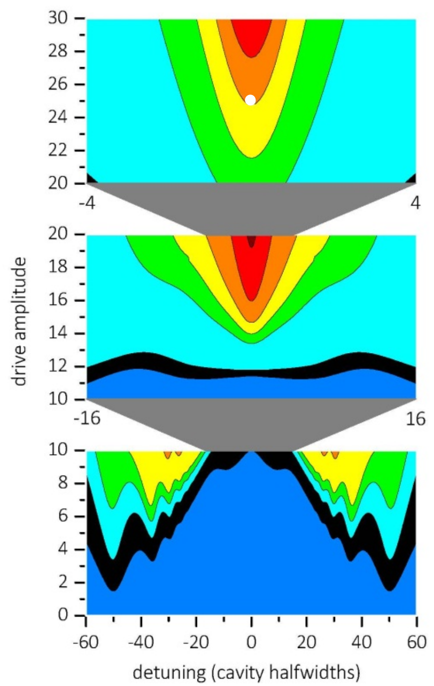
$\gamma/\kappa = 10$

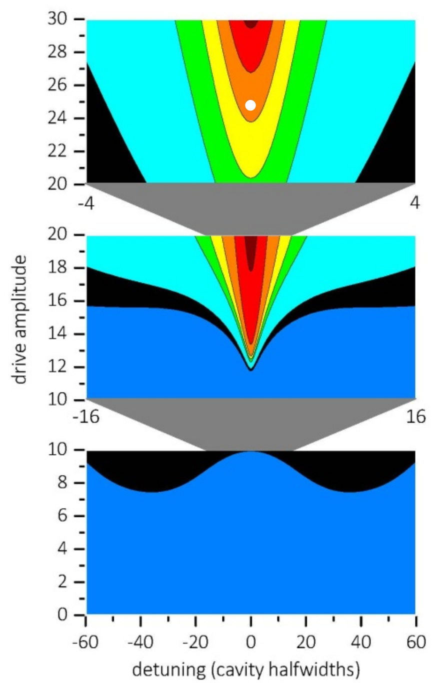
$n_{\text{sat}} = 5\times10^{-3}$

$\gamma/\kappa = 100$

$n_{\text{sat}} = 5\times10^{-1}$

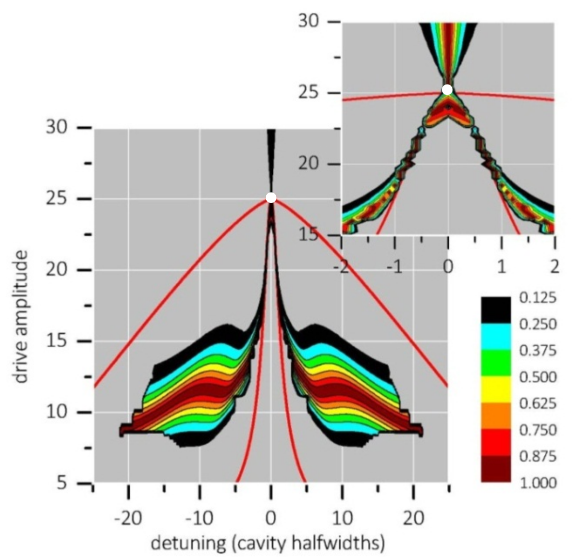


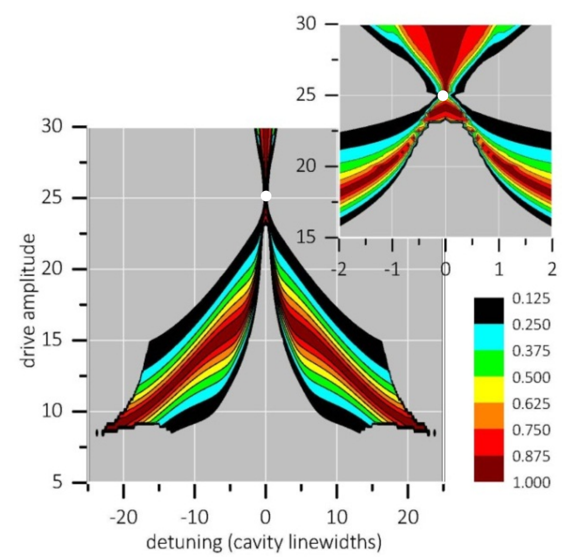




$\gamma/\kappa = 0$
$n_{\text{sat}} = 0$

$\gamma/\kappa = 0.1$
$n_{\text{sat}} = 5{\times}10^{-7}$



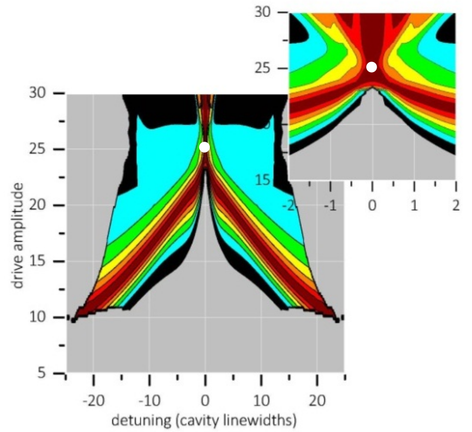


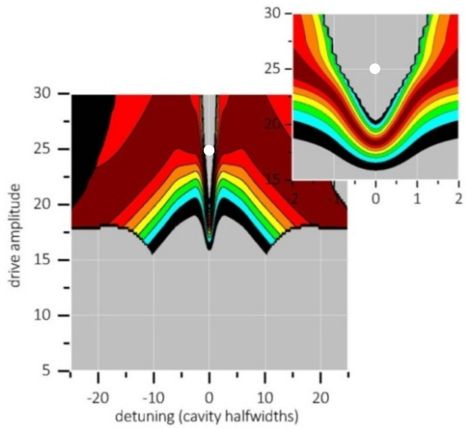
MEAN-FIELD

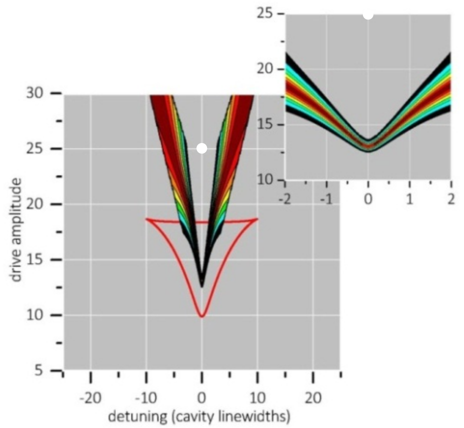
$\gamma/\kappa = 1.0$
$n_{\text{sat}} = 5\times10^{-5}$

$\gamma/\kappa = 10$
$n_{\text{sat}} = 5\times10^{-3}$

$\gamma/\kappa = 100$
$n_{\text{sat}} = 5\times10^{-1}$







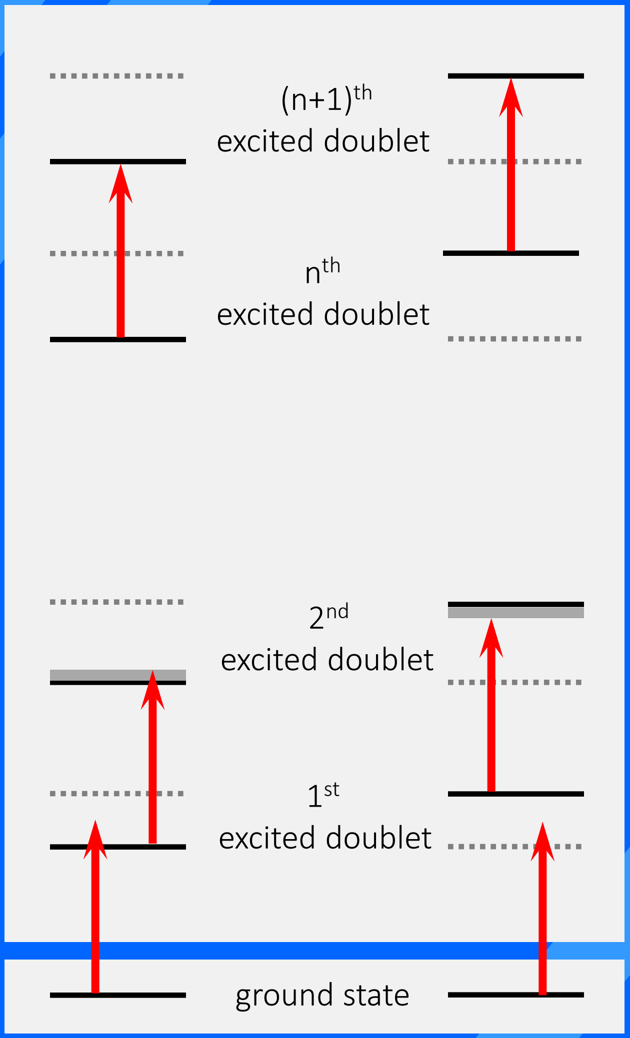
MEAN-FIELD

$$a|n,L\rangle$$

$$\frac{\sqrt{n}+\sqrt{n\text{-}1}}{\sqrt{2}\,\sqrt{2n\text{-}1}}\,|n\text{-}1,L\rangle+\frac{\sqrt{n}-\sqrt{n\text{-}1}}{\sqrt{2}\,\sqrt{2n\text{-}1}}\,|n\text{-}1,U\rangle$$

$$(|\mathrm{g}\rangle\langle\mathrm{e}|)|n,L\rangle$$

$$\frac{1}{\sqrt{2}}\,|n,L\rangle+\frac{1}{\sqrt{2}}\,|n,U\rangle$$



LADDER SWITCHING